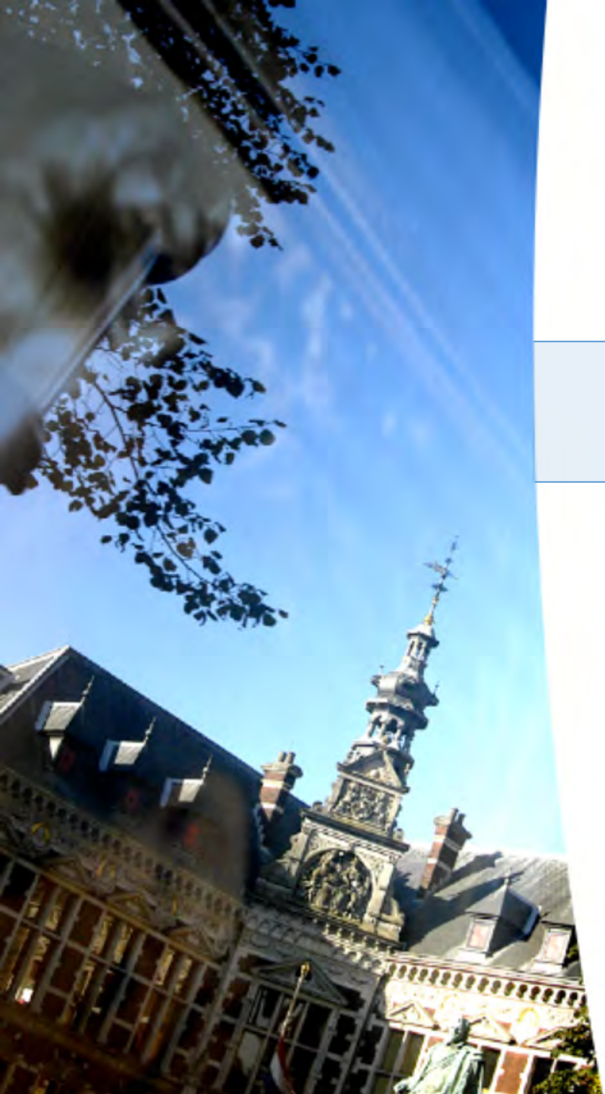


RME 6
CAYMAN ISLANDS

6th International Realistic Mathematics Education Conference
Grand Cayman, Cayman Islands, September 20 – 22, 2018

New developments in Realistic Mathematics Education: the *Beyond Flatland* project

Marja van den Heuvel-Panhuizen
&
Michiel Veldhuis

University of Applied Sciences
iPabo

Freudenthal Group
Freudenthal Institute

Nord University

Reform in the Netherlands started in ~1968

Mechanistic Mathematics Education → **Realistic Mathematics Education**

Focus on procedural skills

Focus on conceptual understanding

2018

mechanistic mathematics education

TAAK 53. 109

1. 20/1480\ 30/2190\ 40/2160\
   50/3400\ 60/4860\ 20/1640\
   90/4680\ 70/2170\ 80/6560\
   70/3710\ 80/5120\ 60/1620\

2.

430	321	212	203	142
19 ×	28 ×	37 ×	46 ×	55 ×

3.

1458	567	2048	2348	738
2057	3296	372	1356	2367
143	25	59	1854	4
17	4647	5788	2973	815

4. *Aftrekken:*

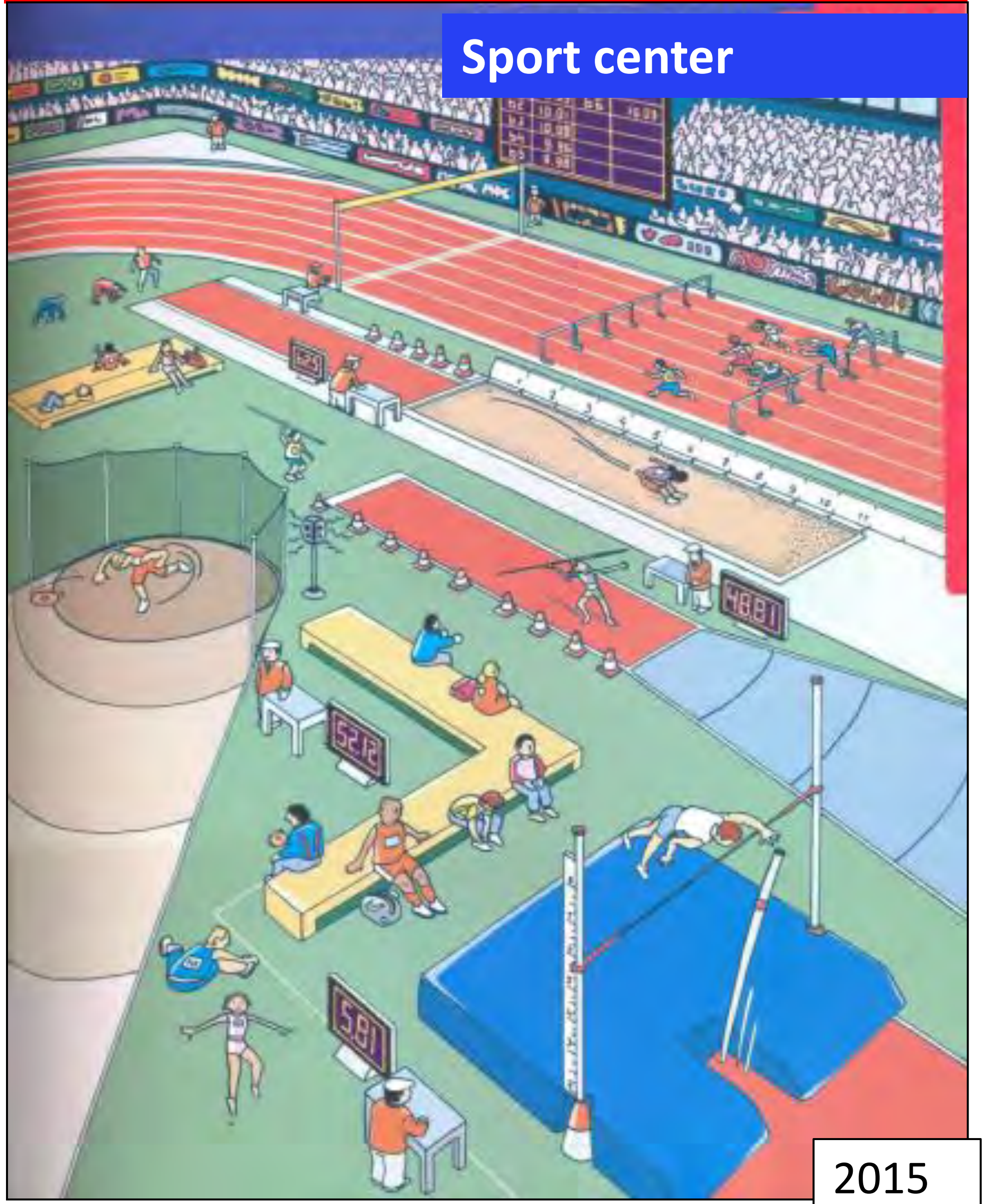
7100	8000	6042	3810	7002
3675	4783	5291	2170	6999

5. Bob, Wim and Koos have f 6 together.
   Bob has f 0.95 Koos has f 3.15 Wim has f …

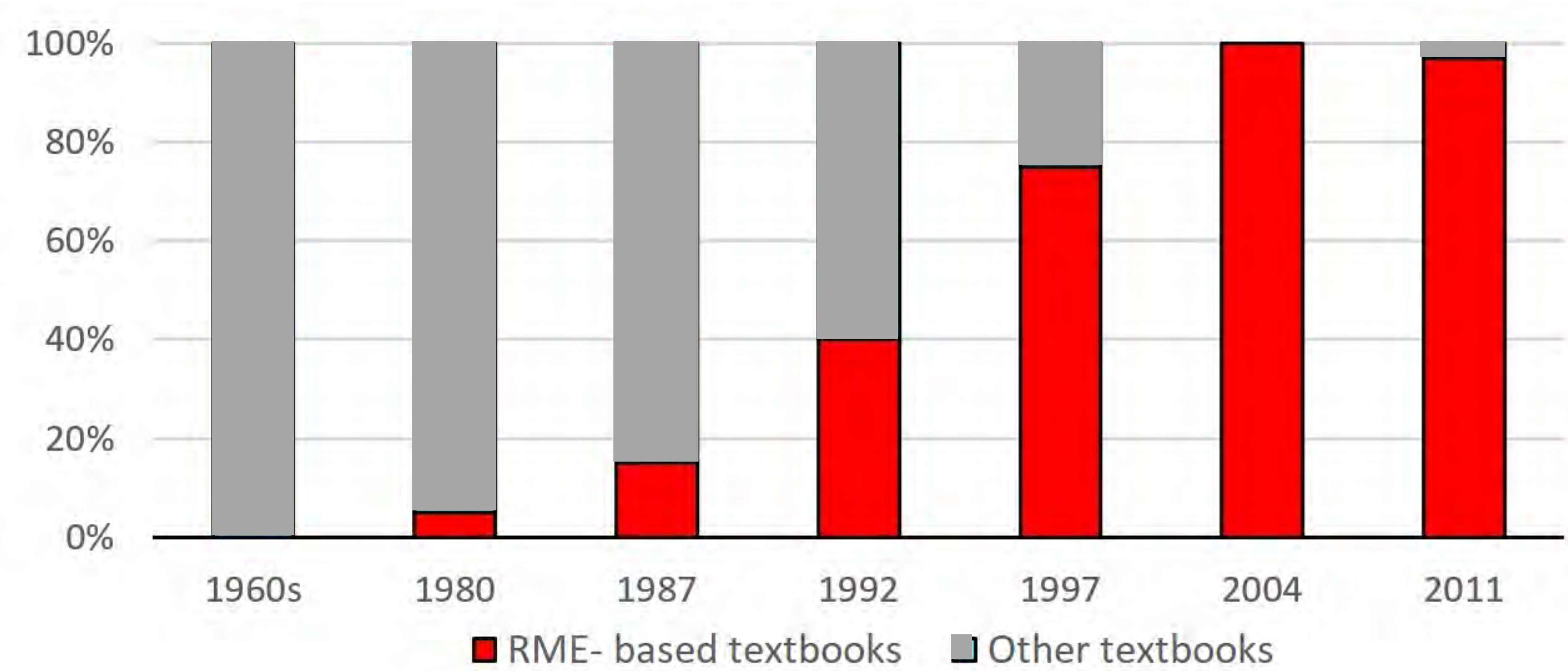
6. 1 jaar = .. mnd. | 26 mnd. = .. jaar + .. mnd.
   3 jaar = .. mnd. | 39 mnd. = .. jaar + .. mnd.
   6 jaar = .. mnd. | 63 mnd. = .. jaar + .. mnd.
   = .. mnd. | 72 mnd. = .. jaar + .. mnd.

1969

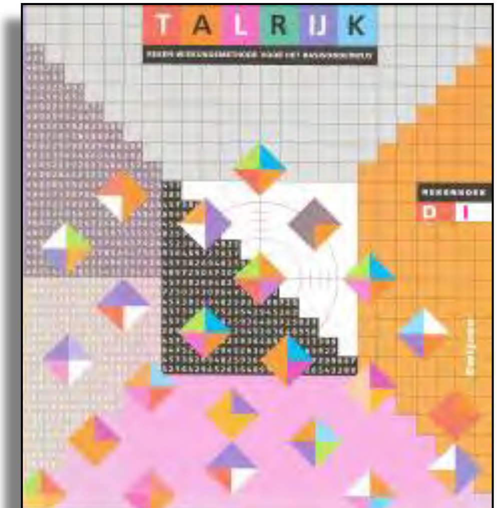
Realistic Mathematics Education



2015

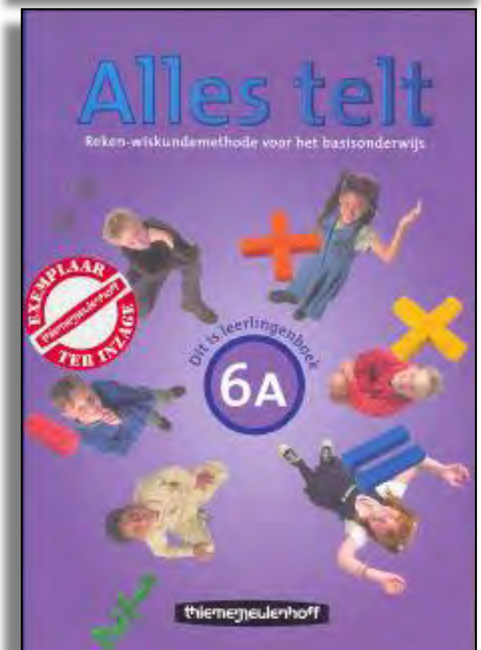

100%
80%
60%
40%
20%
0%
1960s
1980
1987
1992
1997
2004
2011
RME- based textbooks
Other textbooks


Rekenboek A
6
De wereld
in getallen


TALRIJK


RekenRijk
6a
Wolters Noordhoff


LESBOEK
GROEP
6
PLUSPUNT
REKEN-WISKUNDEMETHODE VOOR DE BASISSCHOOL


Alles telt
Reken-wiskundemethode voor het basisonderwijs
EXEMPLAAR
TER INZAGE
Dit is leerlingenboek
6A
thiemeMeulenhoff


Wis en
Reken
Bekadidact

Straightforward task

25 : 5 =
55 : 5 =
105 : 5 =
130 : 5 =
145 : 5 =

In class there are 28 children.
Each child will get a holder with six pencils.
How many pencils need the teachers for this?

Gray area tasks

Pay the exact amount.
Try it in at least five ways.

Puzzle-like task

20 − ▼ = ...

20 + ▼ = ...

20 × ▼ = ...

Together 160

Mediterranean Journal for
Research in Mathematics Education
Vol. 8, 2, 31-68, 2009

Non-Routine Problem Solving Tasks in Primary School Mathematics Textbooks – A Needle in a Haystack

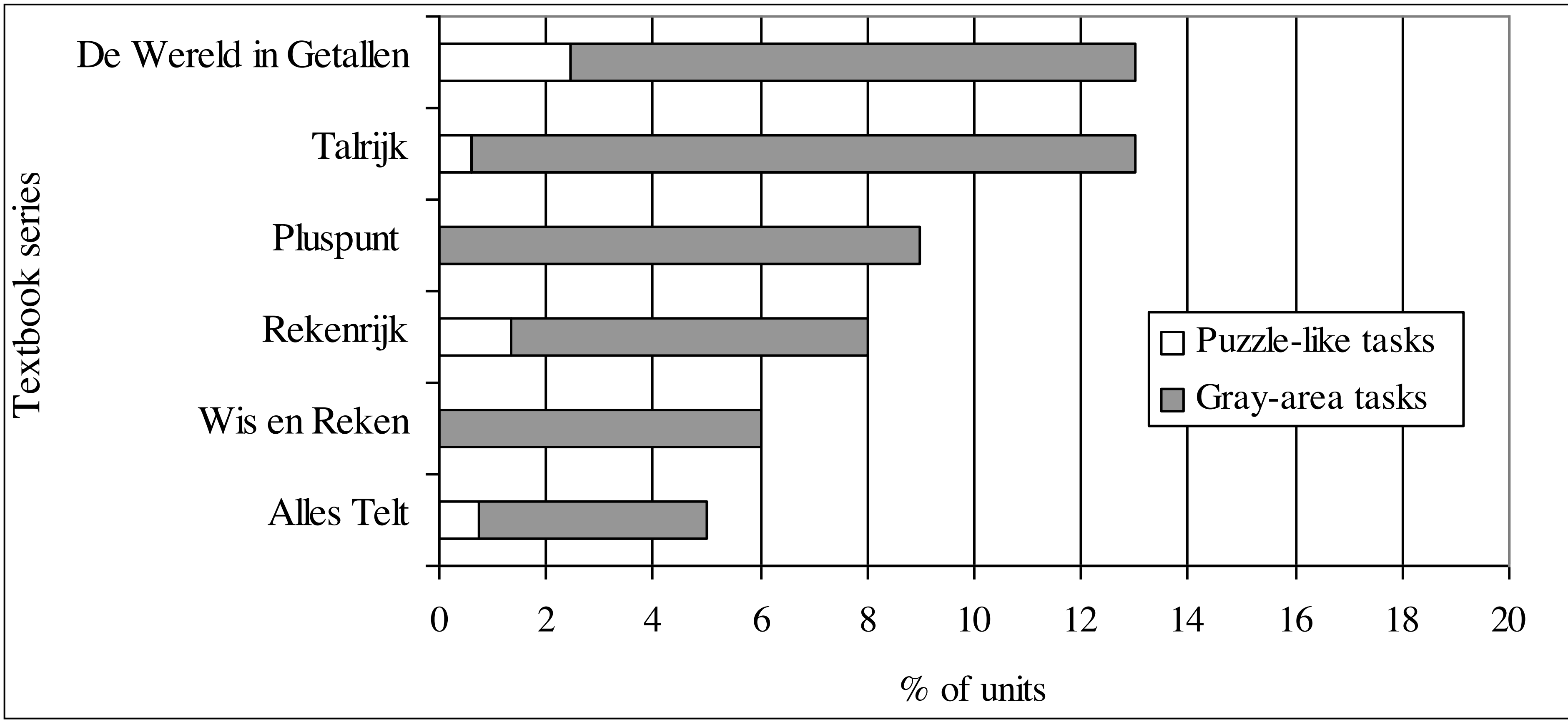
Angeliki Kolovou *, Marja van den Heuvel - Panhuizen ** and Arthur Bakker*

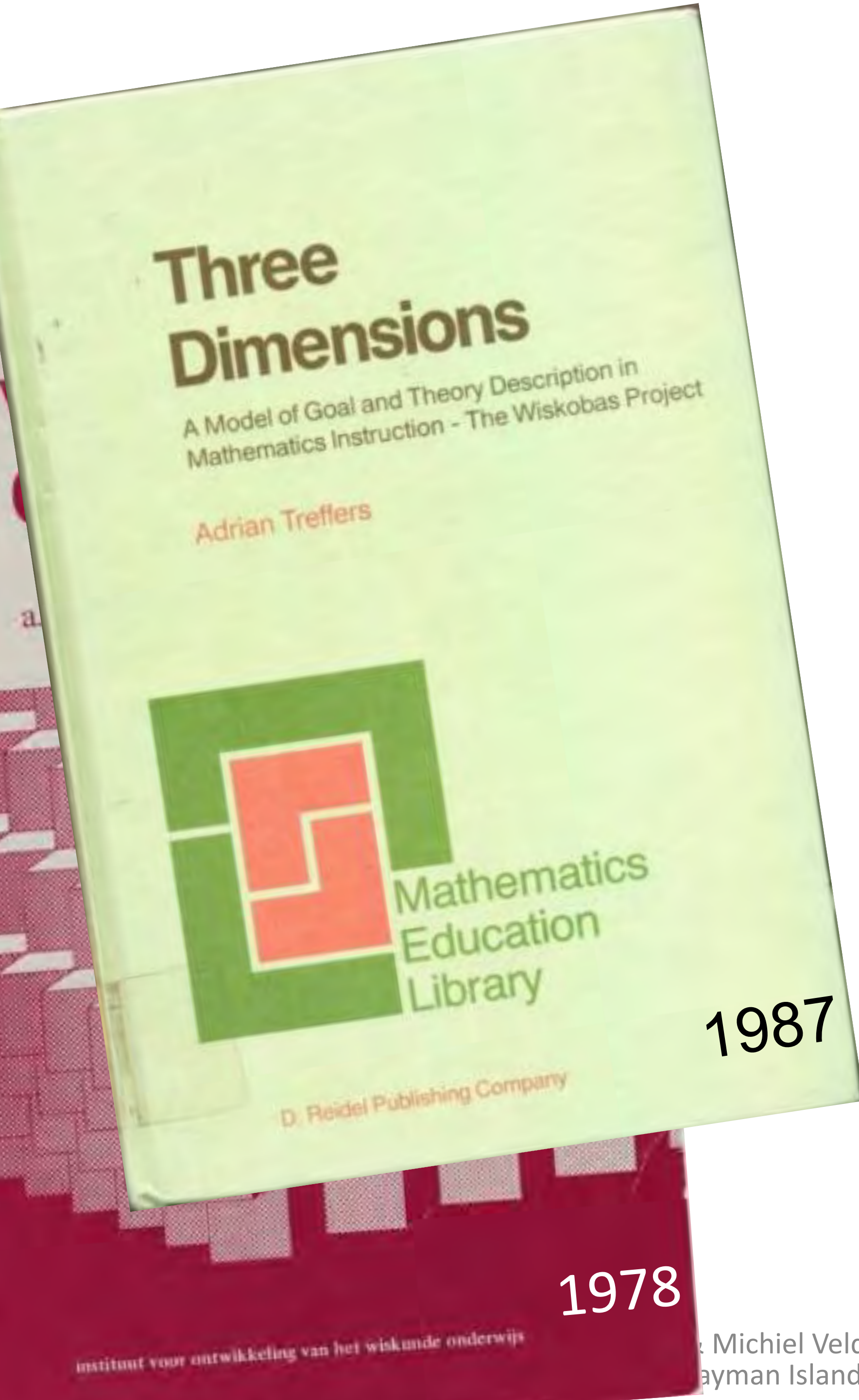
* Freudenthal Institute for Science and Mathematics education, Utrecht University, the Netherlands

** Freudenthal Institute for Science and Mathematics education, Utrecht University, the Netherlands and IQB, Humboldt University, Berlin, Germany

ABSTRACT: *In this paper, we report on a study in which we investigated the nature o*
numerical problem solving tasks as presented in primary school mathematics textbook
in the Netherlands. Although several factors influence what mathematics teachers tea
children, there is much evidence that the curriculum and the textbooks are importa
determinants of what children are taught and what they learn. Contradicting resu
... poor performances of Dutch fourth graders on a test on mathemati
... reasons for this textbook analysis study.
... textbook series.

Results textbook analysis





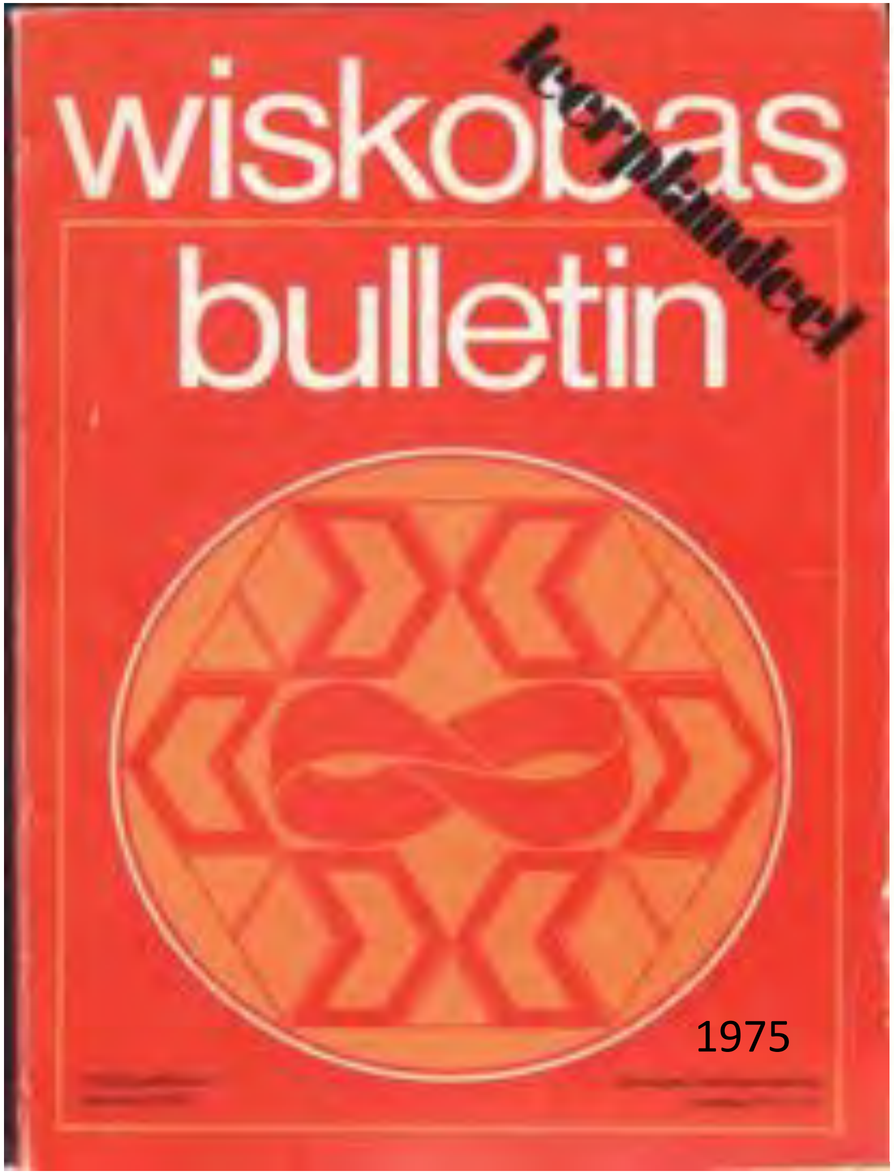
The new **goals** of Wiskobas were

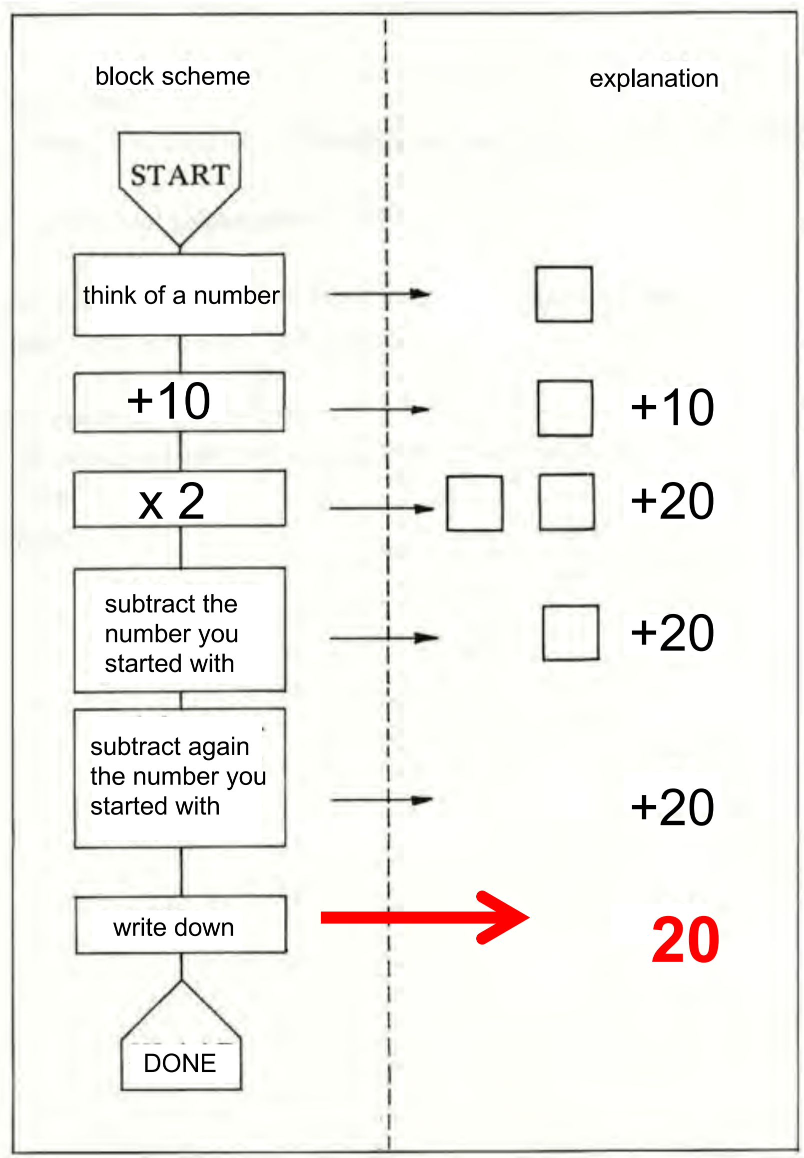
- mathematising
  - generalising
  - proving
  - schematising
  - symbolising
  - using models

and they covered the **subject areas**

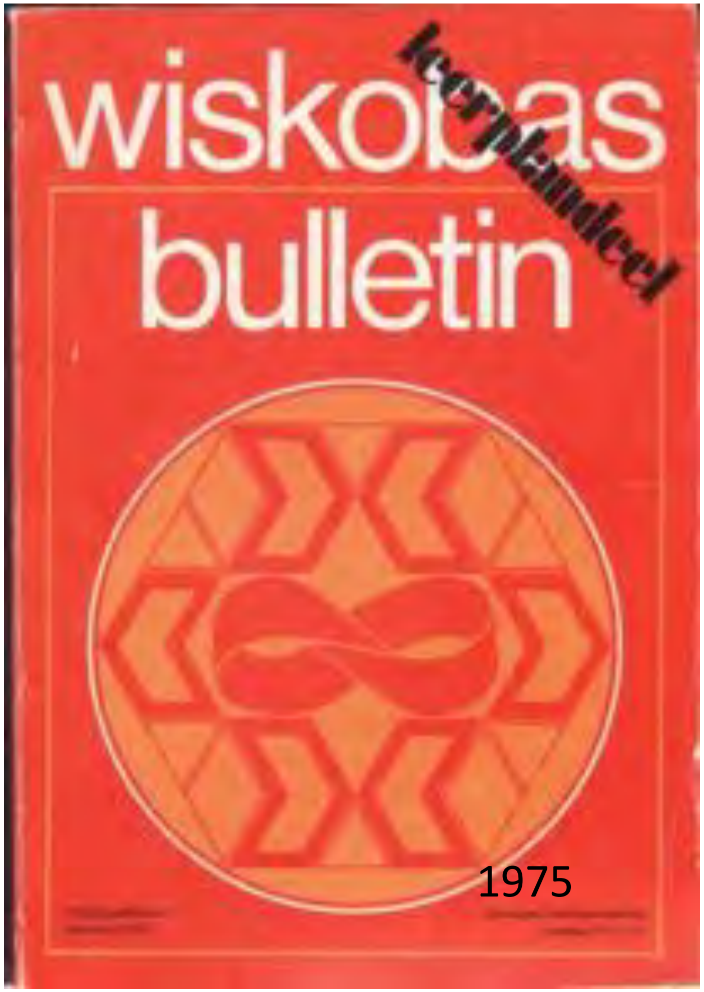
- arithmetic
- measurement
- geometry
- probability and statistics
- relations and functions
- language and logic

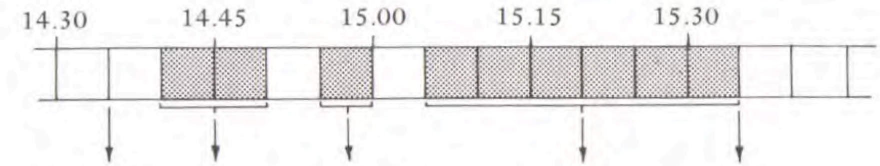
Block schemes to solve equations





Time-distance graphs with a story





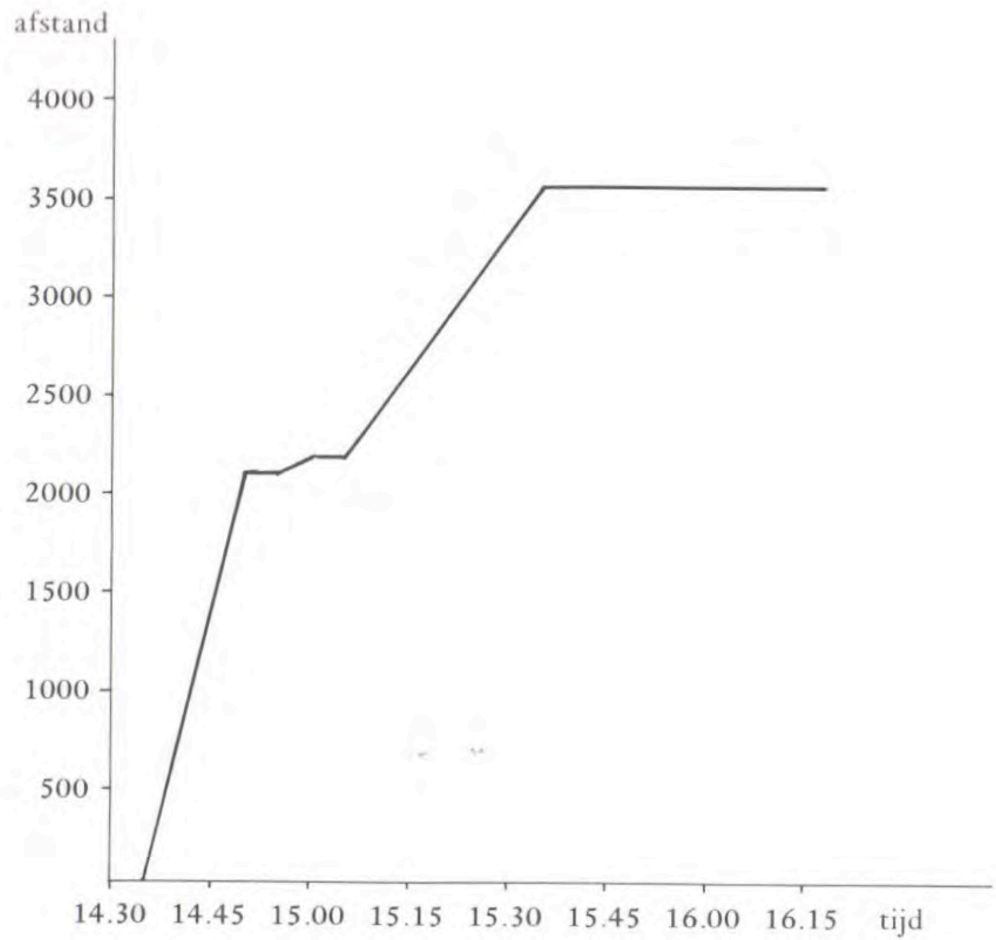
arrival train

bus to city center

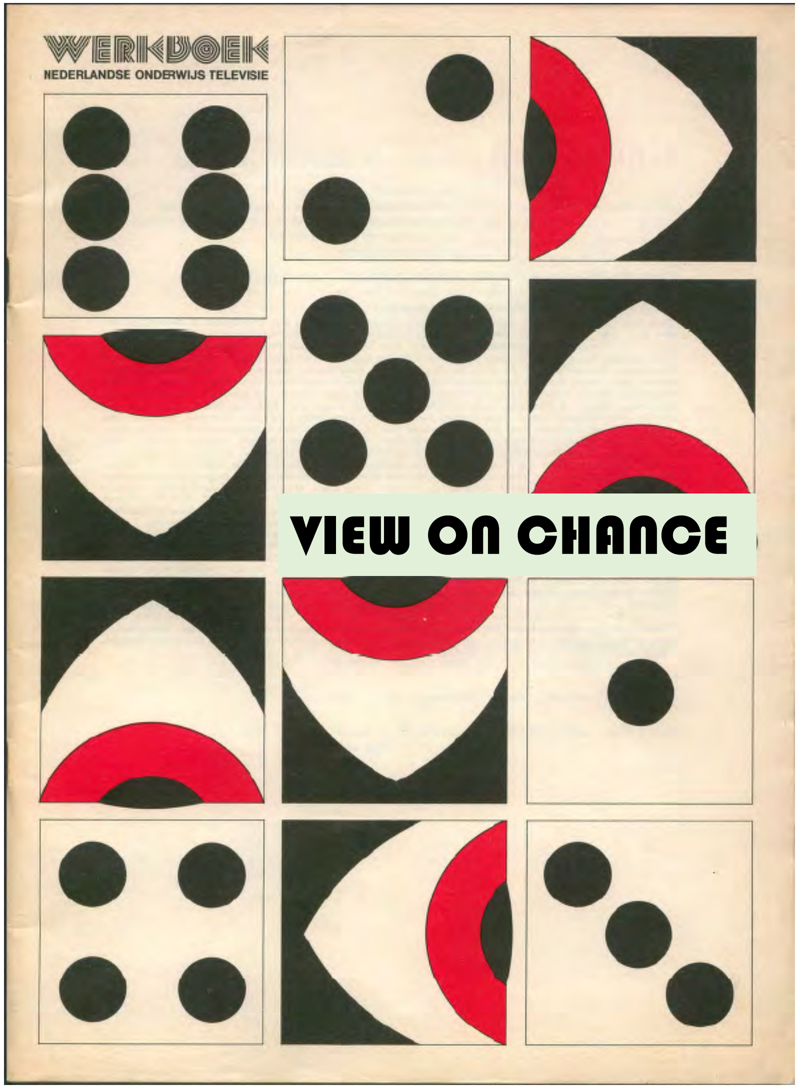
visit to city hall

walk

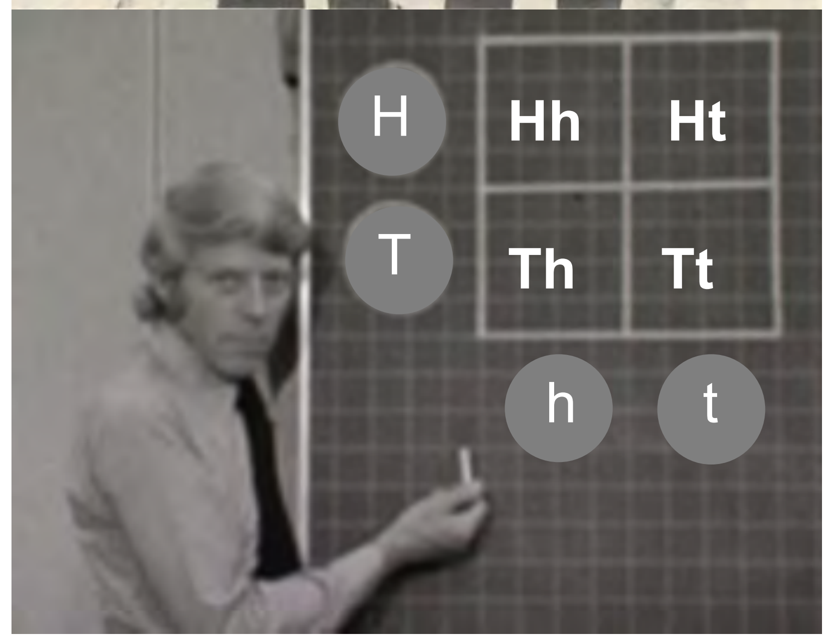
destination



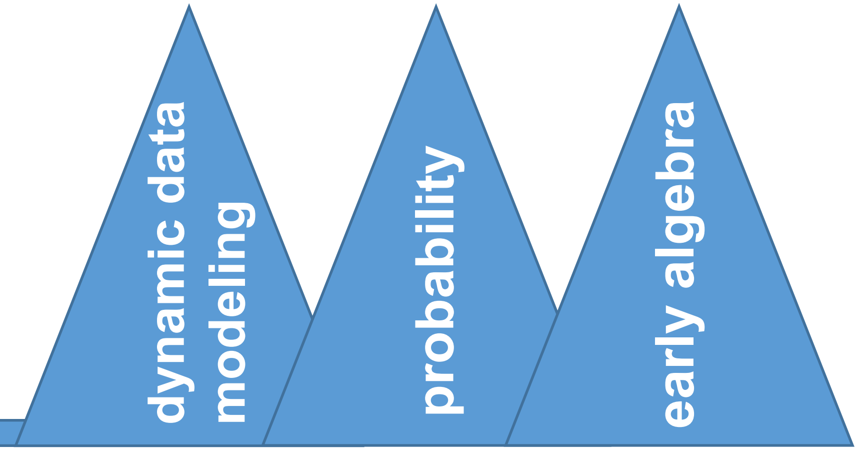
Reasoning about probability



Kijk op kans
Janssen & Goffree, 1972/1973



Beyond FLATLAND in primary school mathematics education



Netherlands Initiative for Education Research

4-year project: 2015-2019

Senior staff

UU	**IPN**
Marja van den Heuvel-Panhuizen	Aiso Heinze
Michiel Veldhuis	Anke Lindmeier
Paul Leseman	
Michiel Veldhuis	
Jan Boom	

PhD students

Carolien Duijzer

Mara Otten

Realistic Mathematics Education

Theoretically enhanced by

- activity principle
- reality principle

- Embodiment theory

- level principle
  * various levels of understanding
  * progressive schematization
  * models as bridges

- Representational re-description theory

- intertwinement principle
- interactivity principle
- guidance principle

- Variation theory

- Our **sensori-motor system** has an important role in developing **conceptual understanding**
- The same neural substrate used in imagining is used in understanding

(Gallese & Lakoff, 2005)

- Embodiment theory

- Representational re-description theory

- Variation theory

- Our **sensori-motor system** has an important role in developing **conceptual understanding**
- The same neural substrate used in imagining is used in understanding

(Gallese & Lakoff, 2005)

“human **ideas** . . . are organized in vast (mostly unconscious) conceptual systems **grounded in physical, lived reality**”

(Núñez, Edwards, & Matos, 1999, p. 50)

- Embodiment theory

- Representational re-description theory

- Variation theory

- Embodiment theory

The RR theory describes the **development of representations**, which can bring students to higher levels of thinking.
The initial implicit, **embodied knowledge**, is in **a next step re-described in verbal or other types of symbolic representations** and, as such, becomes available for explicit verbal-symbolic reasoning and explicit hypothesis-led experimentation.

(Karmiloff-Smith, 1992)

- **Representational re-description theory**

- Variation theory

- Embodiment theory

- Representational re-description theory

- Variation theory

A **necessary condition for learning** is the possibility **to experience variation** and distinguish between what changes and what remains invariant.

(Marton & Booth, 1997; Marton & Pang, 2013)

Being able to **discover structure** and to identify patterns is considered the **essence of mathematics**

(Watson & Mason, 2006)

Therefore, variation theory is considered a **powerful design principle** for mathematics education

(e.g. Sun, 2011; Li, Peng & Song, 2011)

Beyond FLATLAND in primary school mathematics education

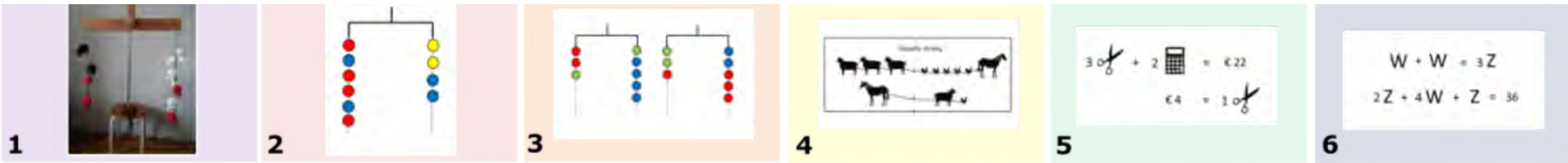
dynamic data modeling

probability

early algebra

How to teach EARLY ALGEBRA

Key components of the Flatland teaching sequence for EARLY ALGEBRA



Focus on: *Algebraic reasoning with linear equations*

More specifically: *Reasoning with, and about, unknowns using algebraic strategies*

Context: *Working with a hanging mobile*

Embodiment: *Experience of balance - equality*

Structure of the Flatland teaching sequence for EARLY ALGEBRA

1. Reactivating the concept of equality; informally use algebraic strategies
2. Eliciting algebraic strategies of restructuring and isolation
3. Eliciting algebraic strategies of isolation and substitution
4. Applying algebraic strategies in a different context: tug-of-war
5. Applying algebraic strategies in a different context to find values
6. Applying algebraic strategies in a formal context to find values

Physical hanging mobile(s)

Other contexts

One equation

System of equations

Relations between unknowns

Values of unknowns

Informal notations

Towards more formal notations

Reactivating the concept of equality

What can you do to keep the hanging mobile straight?

Reactivating the concept of equality

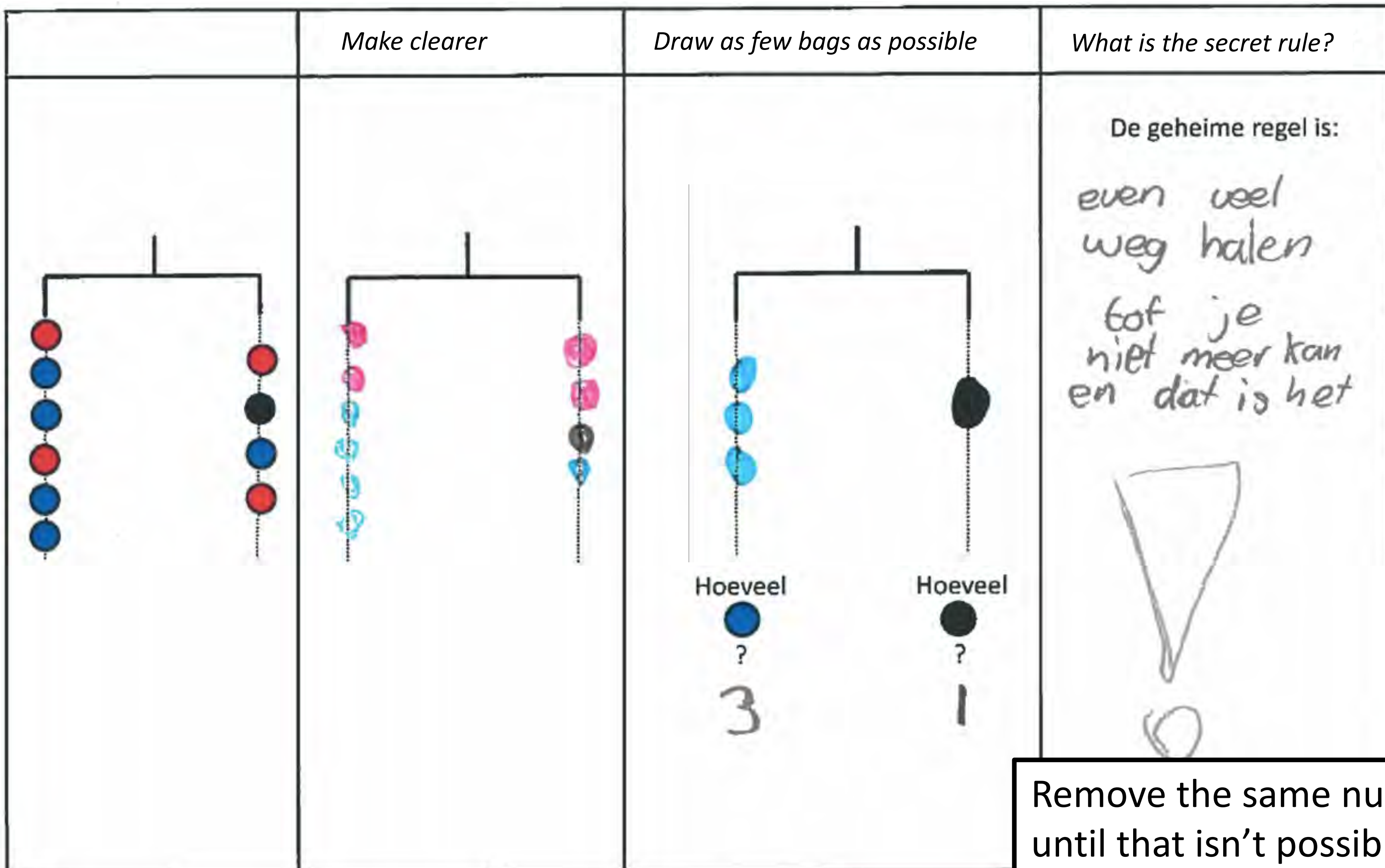
What can you do to keep the hanging mobile straight?

What can you do to keep the hanging mobile straight?

Use the algebraic strategies of

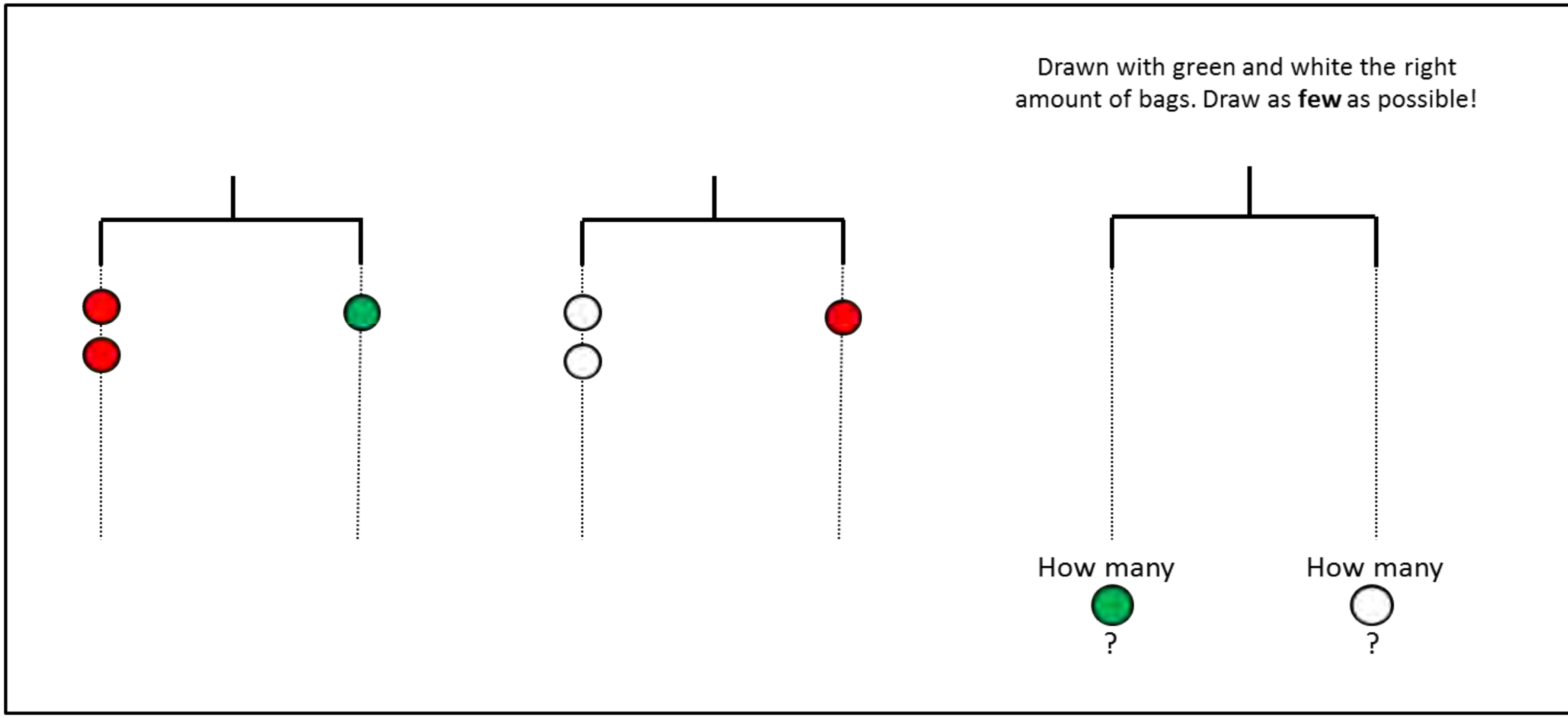
- **Restructuring** by
  - ➢ Changing sides
  - ➢ Changing order of bags on the same side
- **Isolation** by
  - ➢ Taking away similar bags on both sides
  - ➢ Taking away different bags on both sides
- **Substitution** by
  - ➢ Replacing bags by bags of another color

Can you make the hanging mobile clearer and discover the secret rule?



Remove the same number until that isn't possible anymore and that's it!

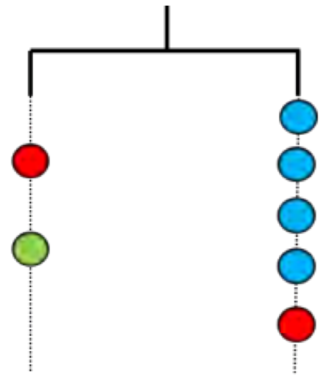
Combine the information from the two given hanging mobiles to find the other relationship



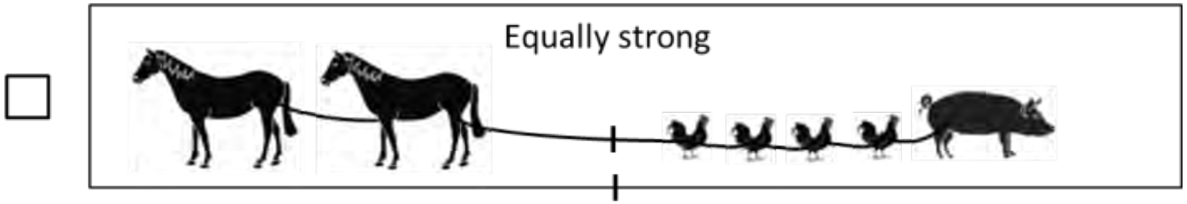
Eliciting algebraic strategies: isolation & substitution

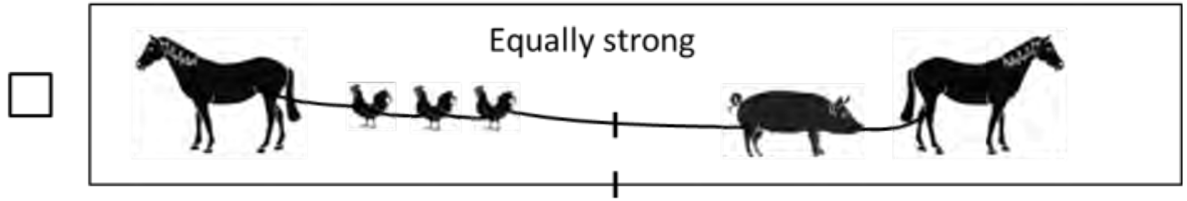
Combine the information from the two given hanging mobiles to find the other relationship

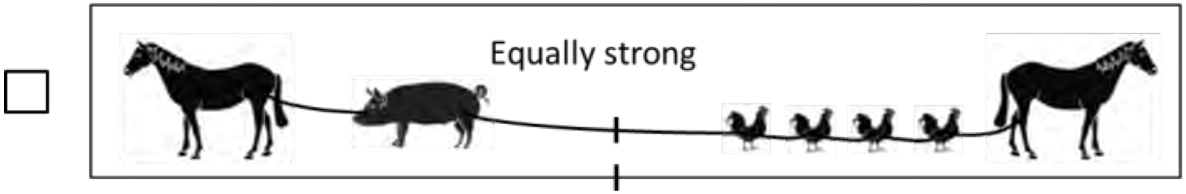
4 Applying algebraic strategies in a different context



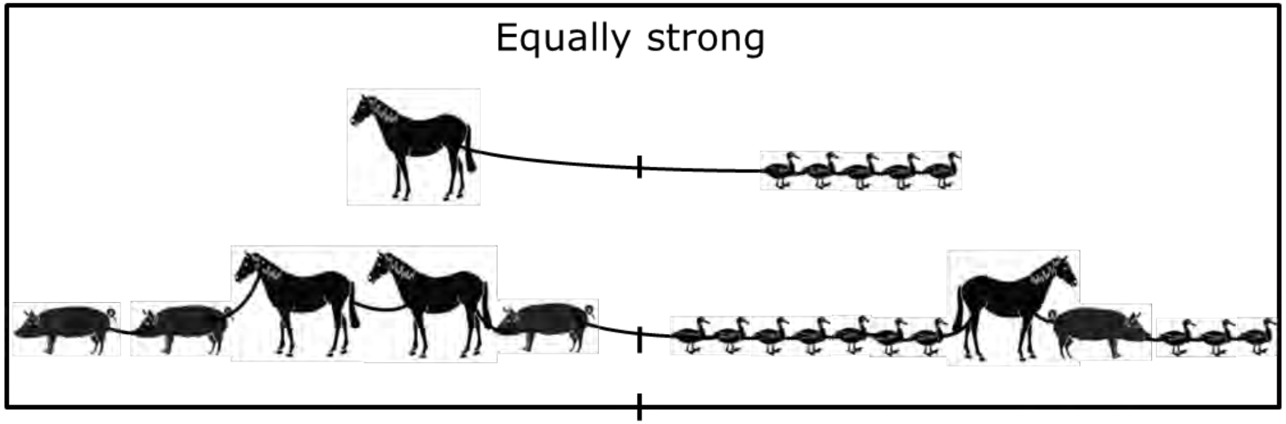
Which hanging mobile fits the tug of war situation?



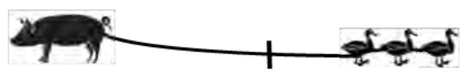




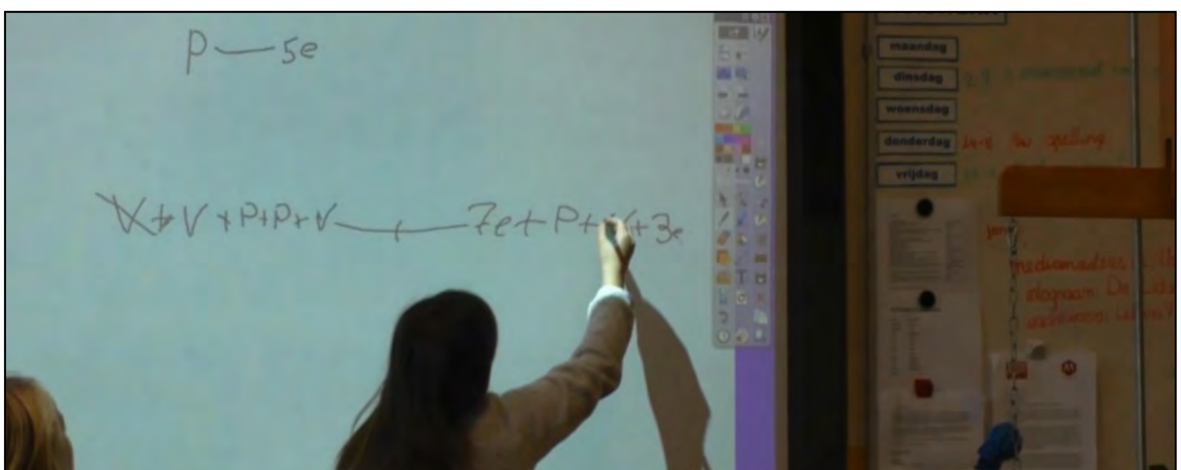
4 Applying algebraic strategies in a different context



Who will win?

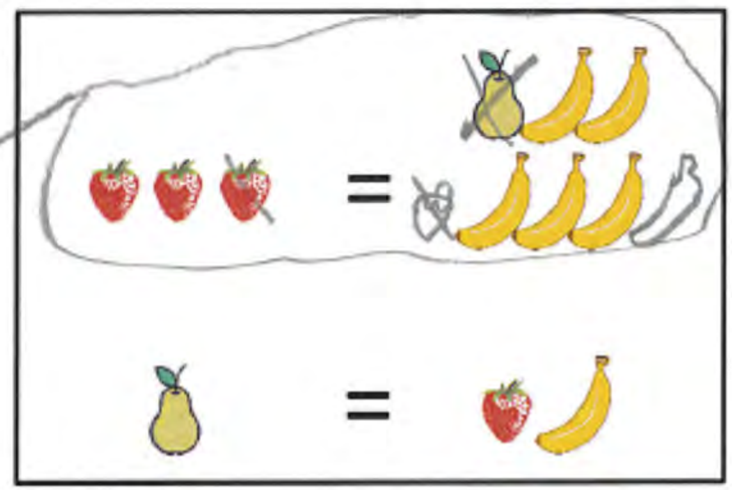


☐ ☐



4 Applying algebraic strategies in a different context

afb 1

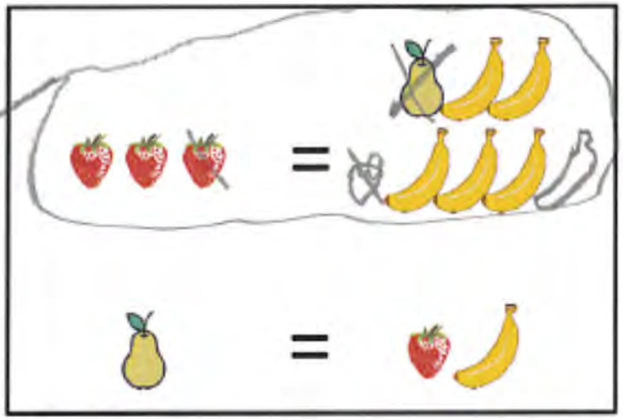


p = peer

a = aardbei
b = banaan

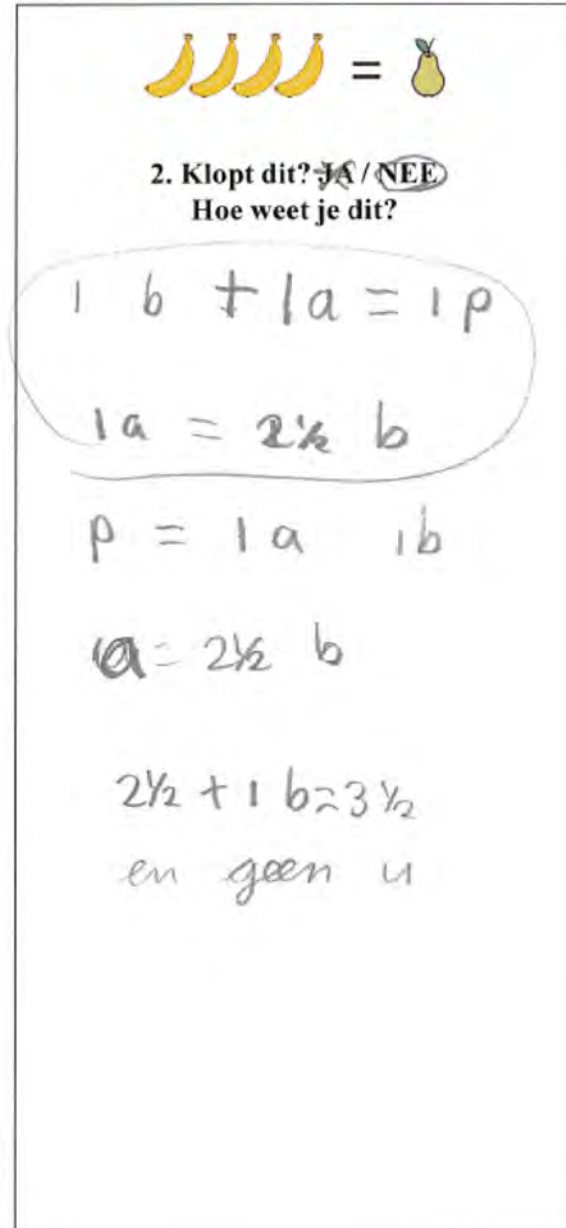
Applying algebraic strategies in a different context

4



1. Klopt dit? JA / NEE
Hoe weet je dit?

bij afb 1 doe je
de peer voor
1 a en 1 b
je streept links
en recht 1 aardbei
door dan heb je
nog 2 a en 5 b
dus dat gedeeld
door 2 =
1 a 2½ b
geen
1 a en 3b

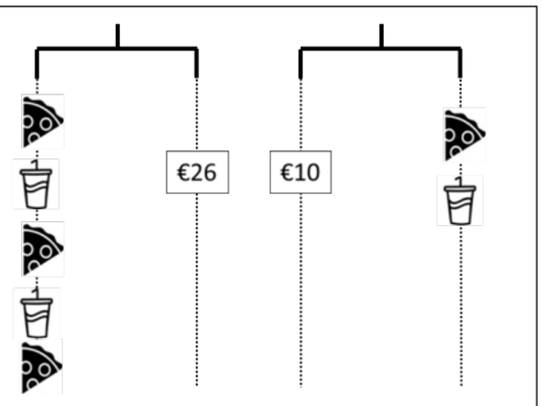


2. Klopt dit? ~~JA~~ / NEE
Hoe weet je dit?

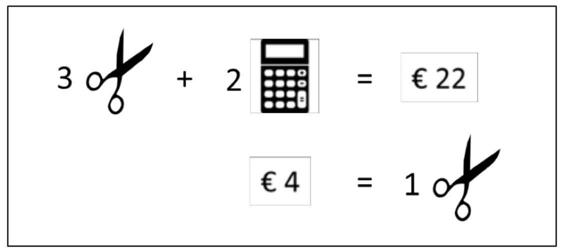
1 b + 1a = 1p
1a = 2½ b
p = 1 a 1b
a = 2½ b
2½ + 1 b = 3½
en geen 4

5

Applying algebraic strategies to find values - informal



Fill in the price



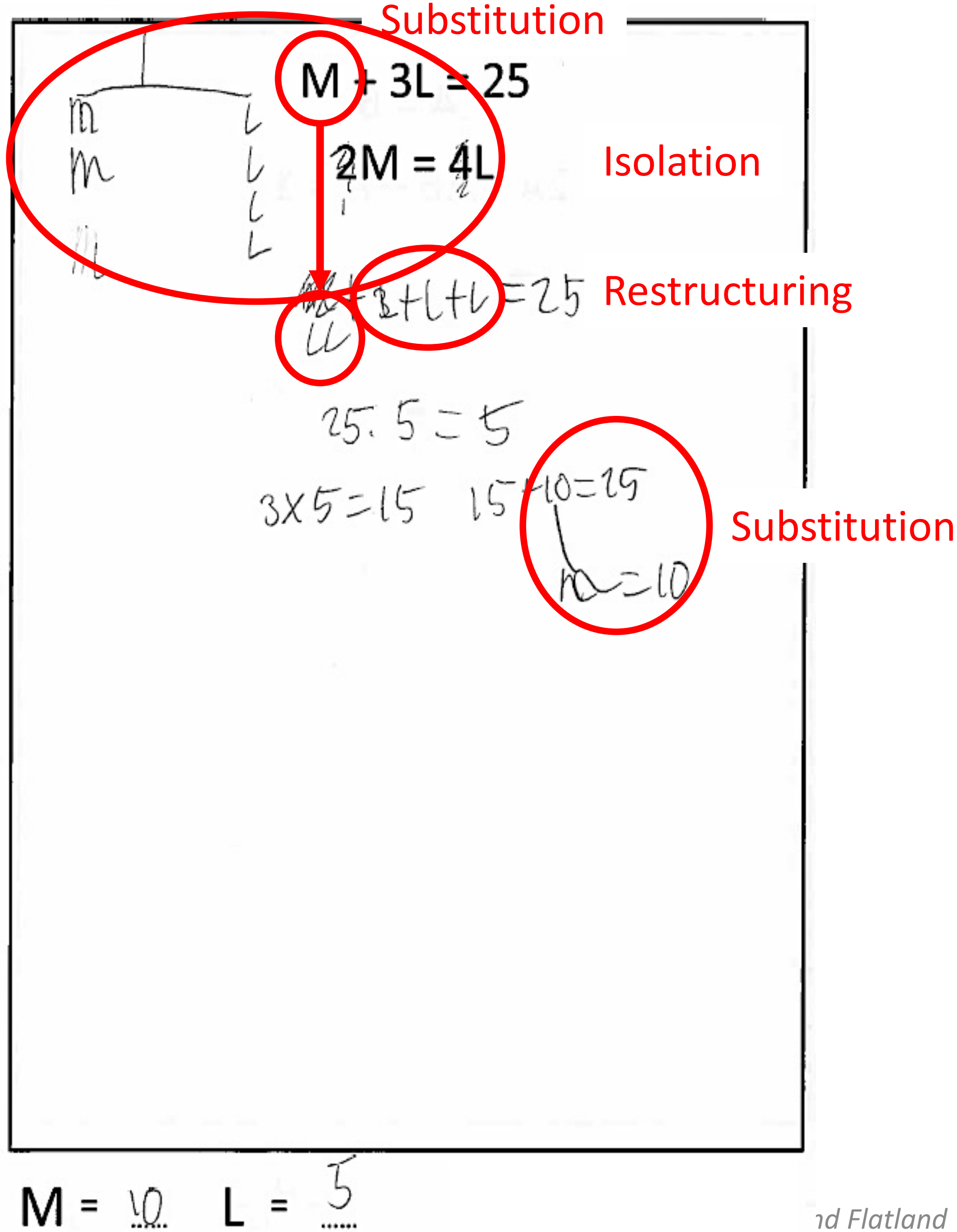
First write this is-equal-to task differently,
then find out what needs to be filled in

$$2P + 4S = 17$$
$$1P + 1S = 5$$

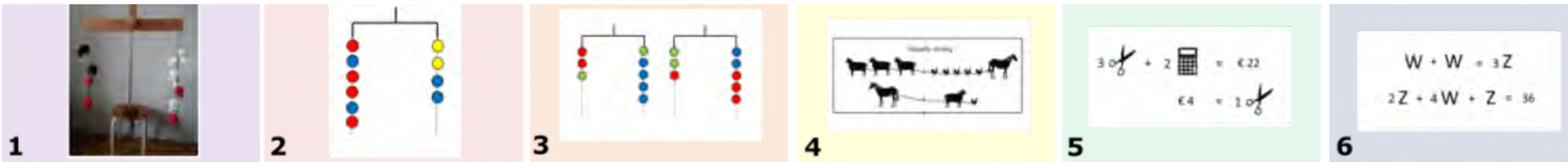
First write this is-equal-to task differently,
then find out what needs to be filled in

P = ... S = ...

Applying algebraic strategies to find values - formal



Key components of the Flatland teaching sequence for EARLY ALGEBRA



Focus on: *Algebraic reasoning with linear equations*

More specifically: *Reasoning with, and about, unknowns using algebraic strategies*

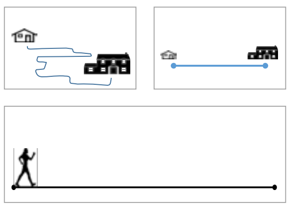
Context: *Working with a hanging mobile*

Embodiment: *Experience of balance - equality*

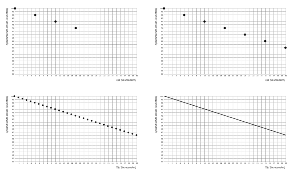
How to teach DYNAMIC DATA MODELING

Key components of the Flatland teaching sequence for DYNAMIC DATA MODELING

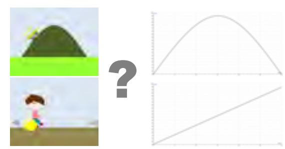
LESSON 1



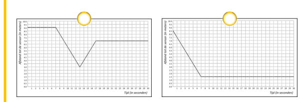
LESSON 2



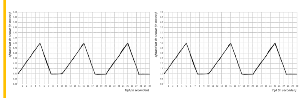
LESSON 3



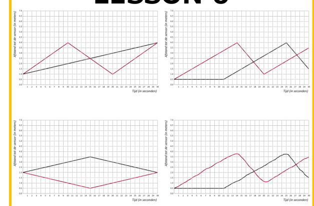
LESSON 4



LESSON 5



LESSON 6



Focus on: *Reasoning about graphical representations of change*

Specifically: *Reasoning about, and interpreting, time-distance-graphs*

Context: *Moving in front of a motion sensor*

Embodiment: *Experience of moving through space – graph (covariation)*

Structure of the Flatland teaching sequence for DYNAMIC DATA MODELING

1. Explore motion: reflecting and representing
2. From discrete to continuous representations of change
3. Continuous graphs of ‘distance to’ (1)
4. Continuous graphs of ‘distance to’ (2)
5. Scaling on the graphs’ axes
6. Multiple movements and their graphical representation

Motion sensor

One graph/ situation

One graph/ multiple situations

Multiple graphs/ situation

Separate time points

Time as continuous variable

Informal graphs

Discrete graphs

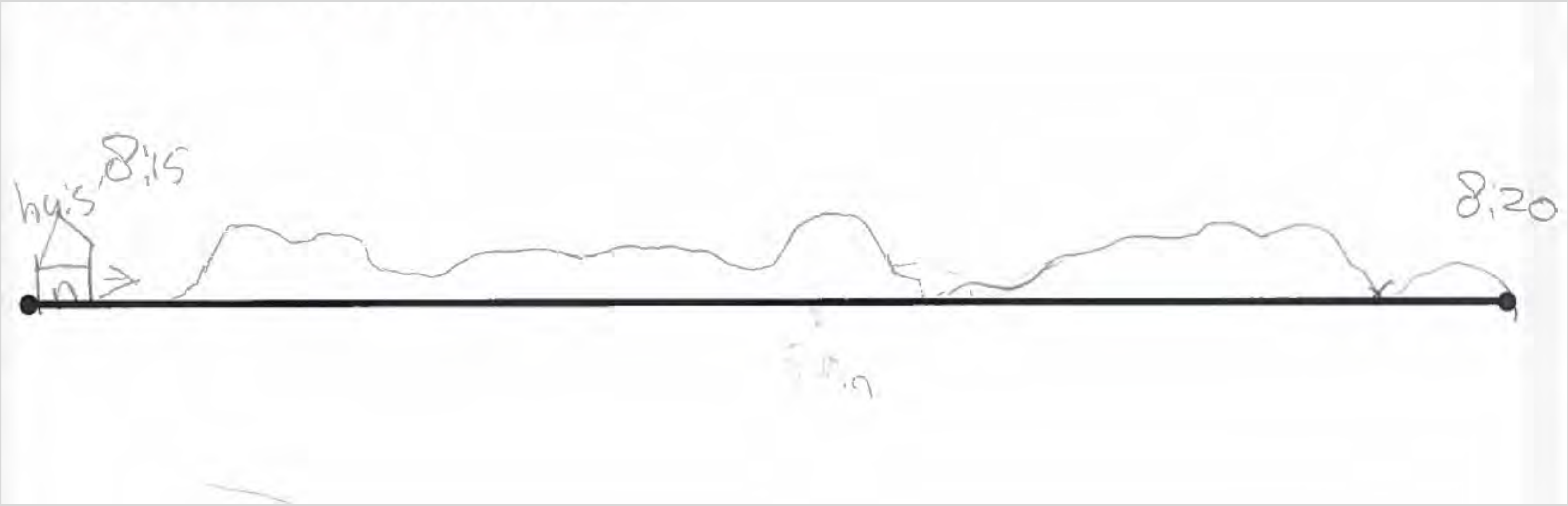
Continuous graphs

Explore motion: reflecting and representing

Represent your trip from home to school

Explore motion: reflecting and representing

Represent your trip from home to school

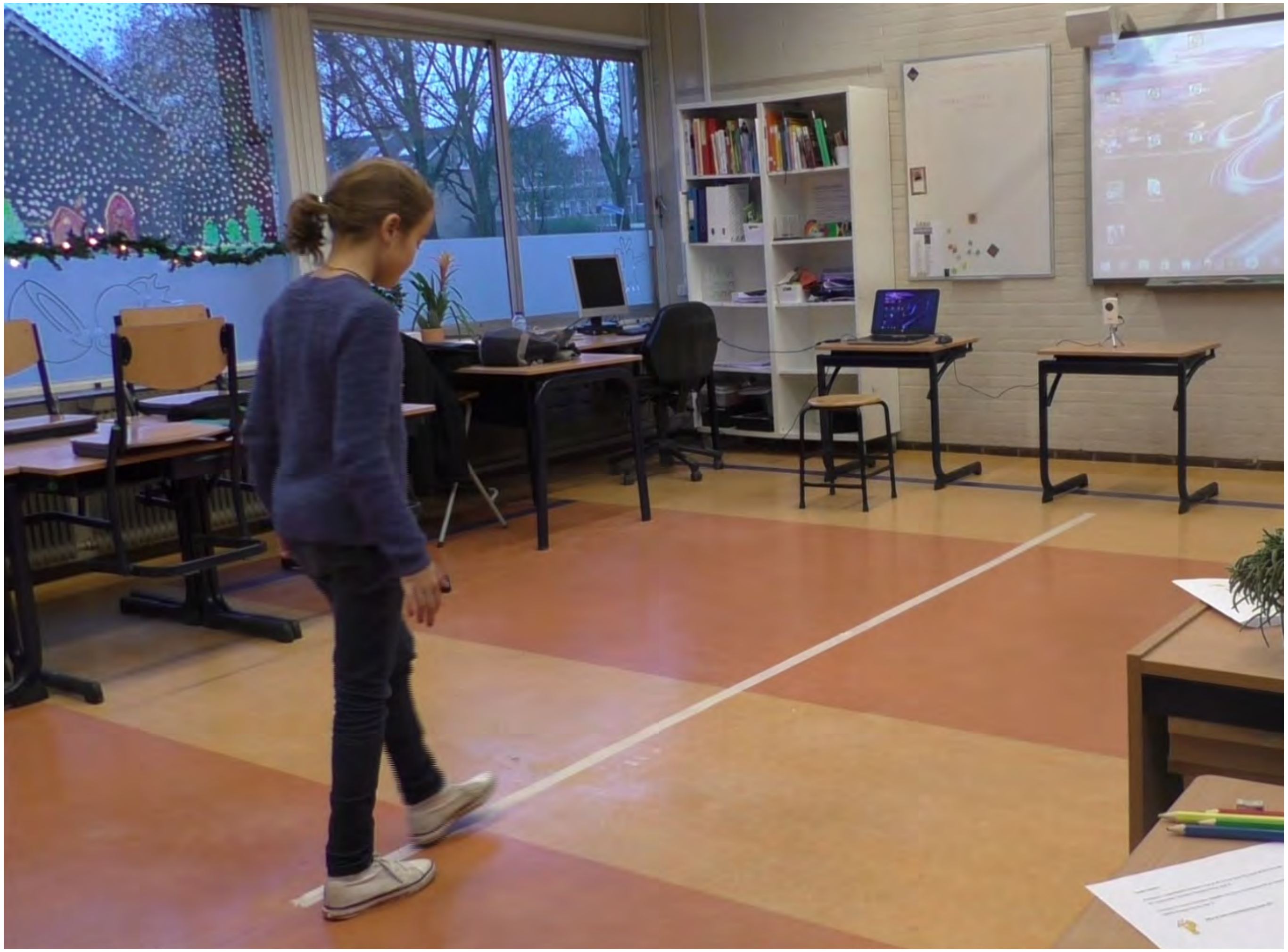


Who arrives first?

***1.** A person walks normally towards the middle and then slowly towards the end.*

***2.** A person walks fast towards the middle, stands still for two seconds, and walks then normally towards the end.*

Who arrives first?



Intruder problem

HET INBREKERSPROBLEEM

Naam: ______________________

In een geheim laboratorium ergens op de wereld worden nieuwe plantsoorten ontwikkeld. Zaadjes van deze planten zijn extreem zeldzaam en heel erg waardevol.

Maar...er is iets vreselijks gebeurd! Midden in de nacht heeft een inbreker zaadjes van een van deze plantsoorten meegenomen! Vanwege de unieke kenmerken van elke plant is het belangrijk dat wordt uitgezocht van welke plant de inbreker zaadjes heeft gestolen. Kun jij de politie helpen dit probleem op te lossen?

Zie je de vier bijzondere planten die in het laboratorium gekweekt worden?

Planten:

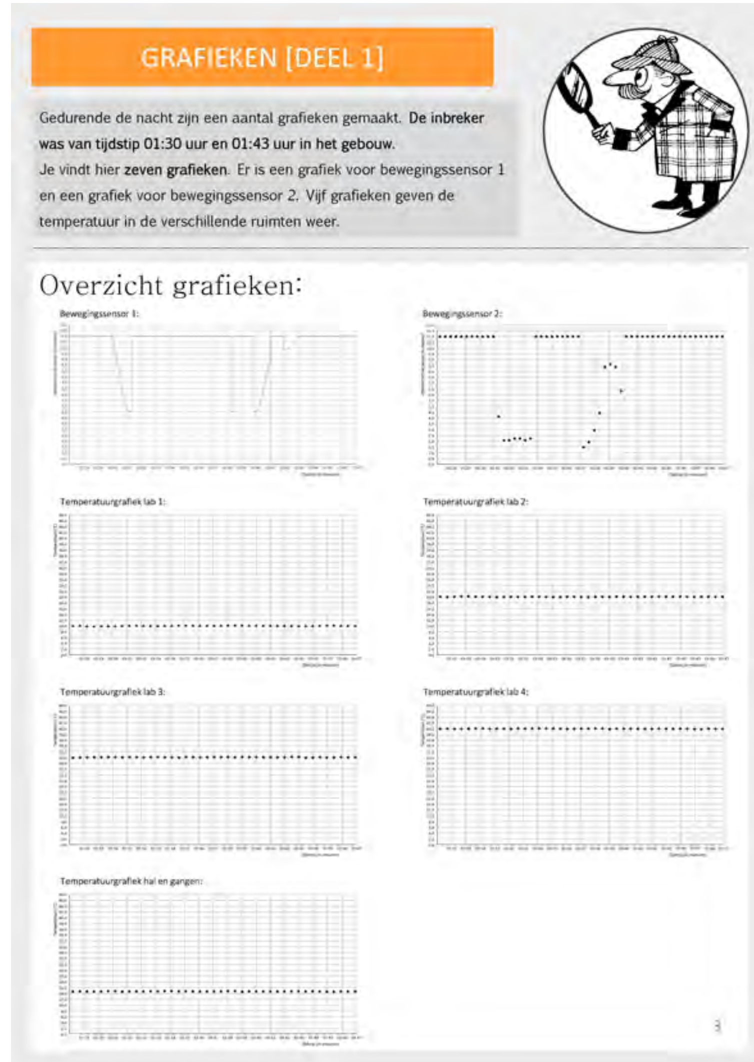
Lab 1
Naam: "Echinops Multiflora Deformis" [Langstelige Kogeldistel]

Lab 2
Naam: "Mirabilis Youlia Apertus" [Open Zevendood]

Lab 3
Naam: "Aconitum Vulparia Magna" [Grote Gele Monnikskap]

Lab 4
Naam: "Elodea Serpyllum" [Wilde Waterpest]

1



GRAFIEKEN [DEEL 1]

Gedurende de nacht zijn een aantal grafieken gemaakt. **De inbreker was van tijdstip 01:30 uur en 01:43 uur in het gebouw.**
Je vindt hier **zeven grafieken**. Er is een grafiek voor bewegingssensor 1 en een grafiek voor bewegingssensor 2. Vijf grafieken geven de temperatuur in de verschillende ruimten weer.

Overzicht grafieken:

Bewegingssensor 1:

Bewegingssensor 2:

Temperatuurgrafiek lab 1:

Temperatuurgrafiek lab 2:

Temperatuurgrafiek lab 3:

Temperatuurgrafiek lab 4:

Temperatuurgrafiek hal en gangen:

3

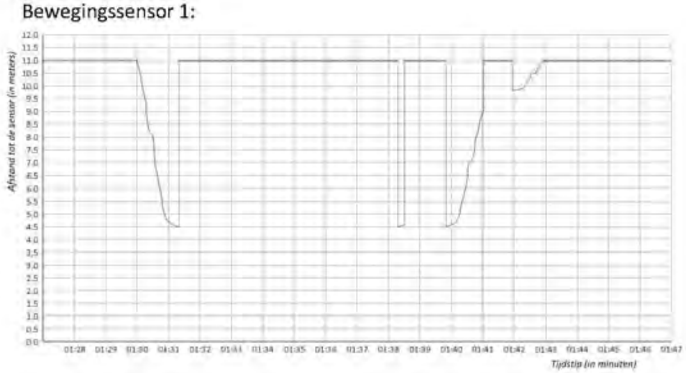
Intruder problem

GRAFIEKEN [DEEL 1]

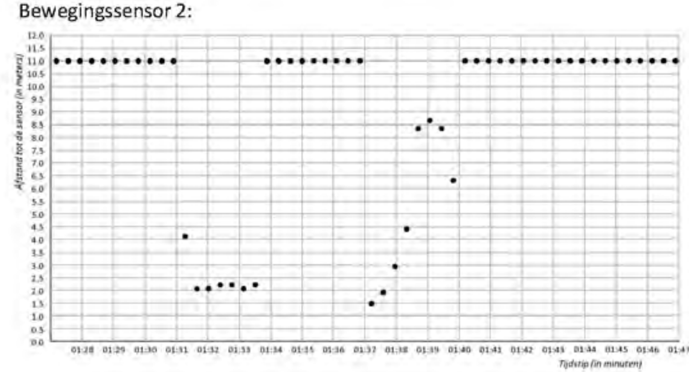
Gedurende de nacht zijn een aantal grafieken gemaakt. **De inbreker was van tijdstip 01:30 uur en 01:43 uur in het gebouw.**
Je vindt hier **zeven grafieken.** Er is een grafiek voor bewegingssensor 1 en een grafiek voor bewegingssensor 2. Vijf grafieken geven de temperatuur in de verschillende ruimten weer.

Overzicht grafieken:

Bewegingssensor 1:



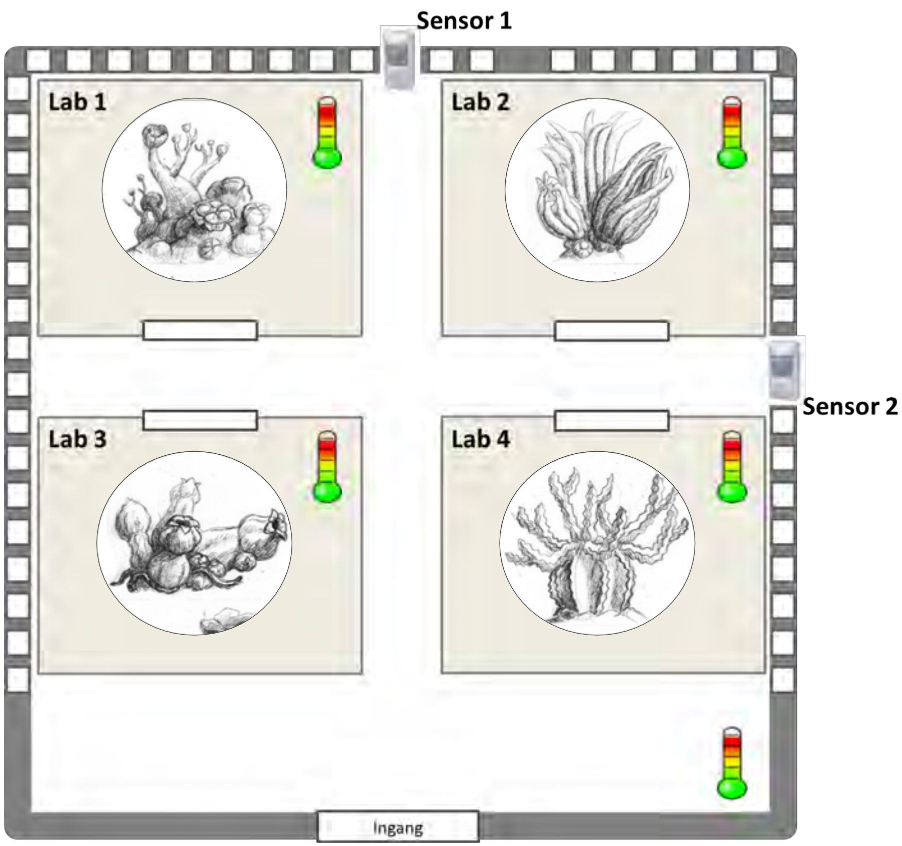
Bewegingssensor 2:



Temperatuurgrafiek lab 1:

Temperatuurgrafiek lab 2:

Intruder problem



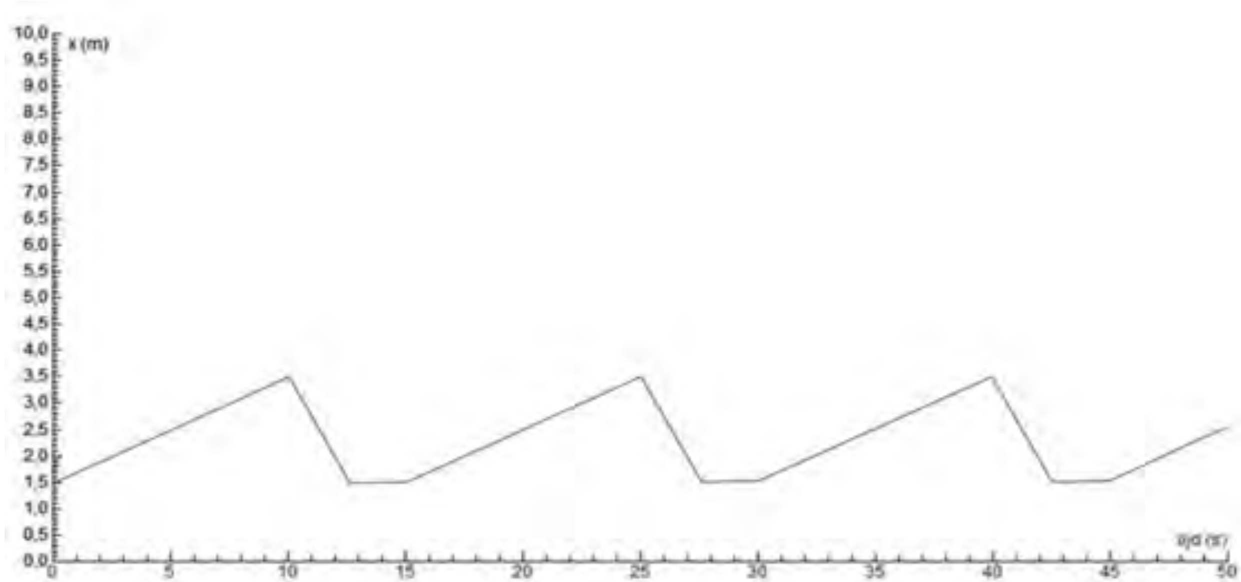
Floor plan

Question:
From which room(s) did the burglar steal some seeds?

Available information:
- Floor plan
- Time the burglar is in the building
- 2 motion graphs
- 5 Temperature graphs

Walking continuous graphs of distance to

Motion sensor



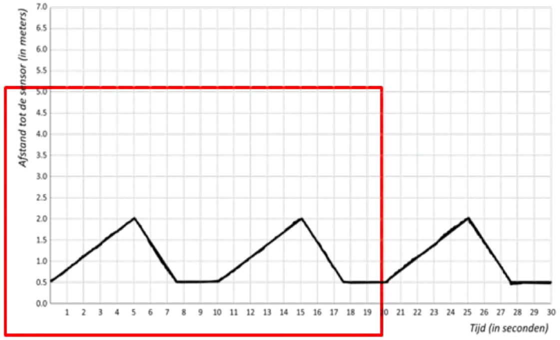
3

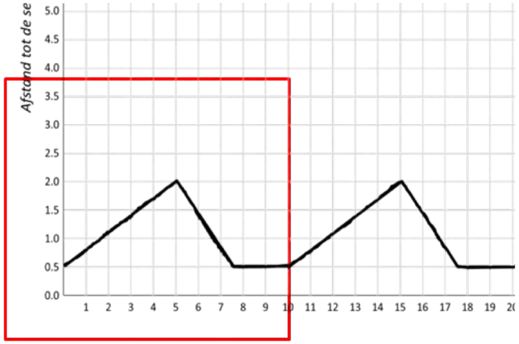
Continuous graphs (1)

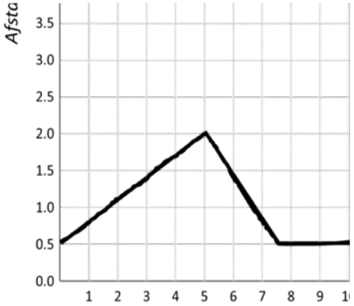
Walking continuous graphs of distance to

Walking continuous graphs with changing speeds

5 Scaling on the graph's axes

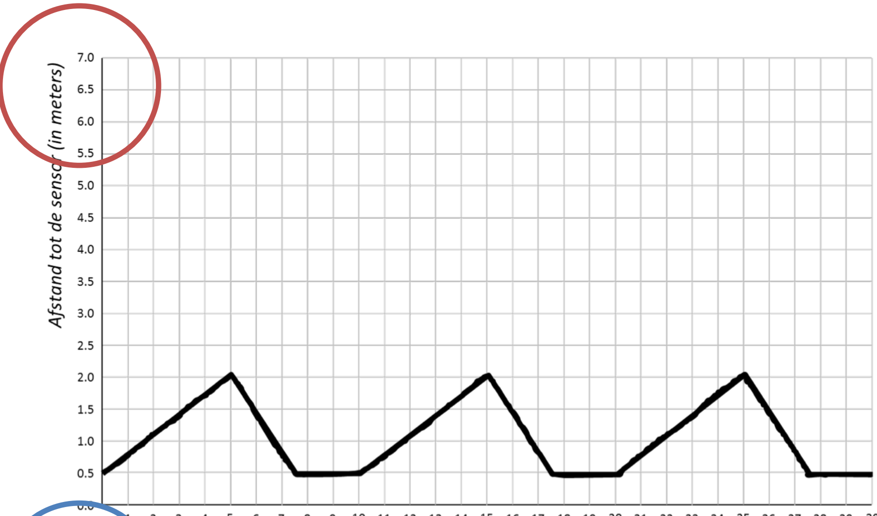


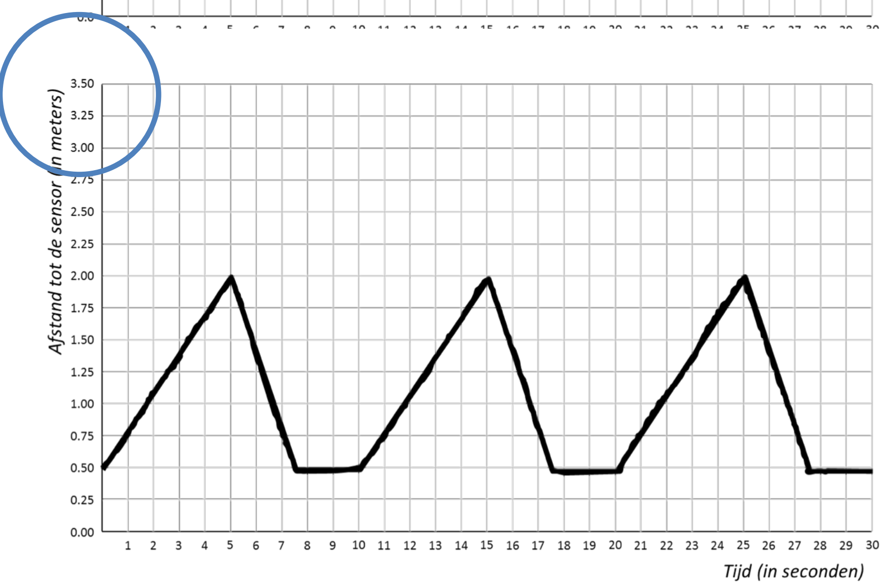


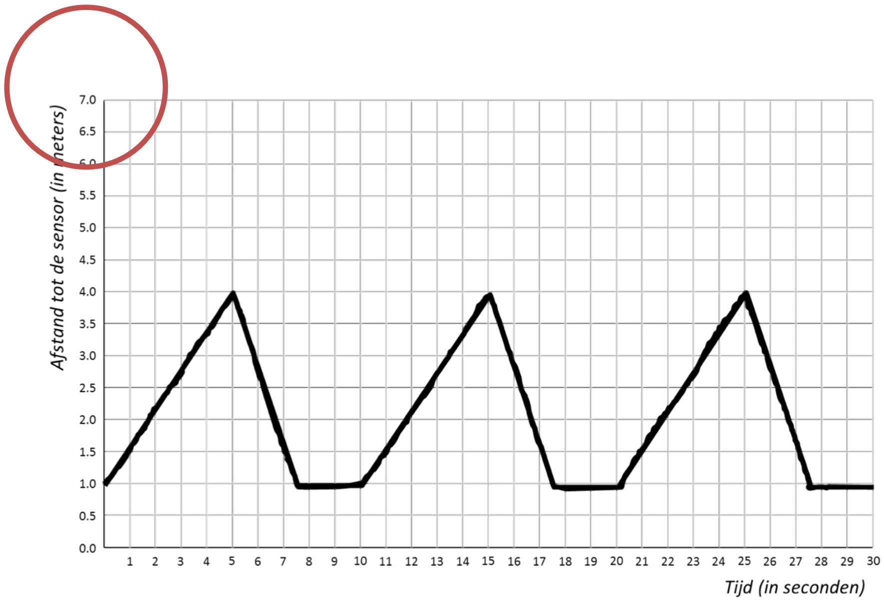


Scaling on the graph's axes

5

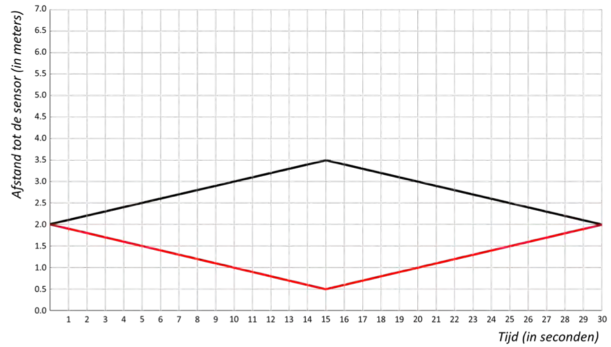




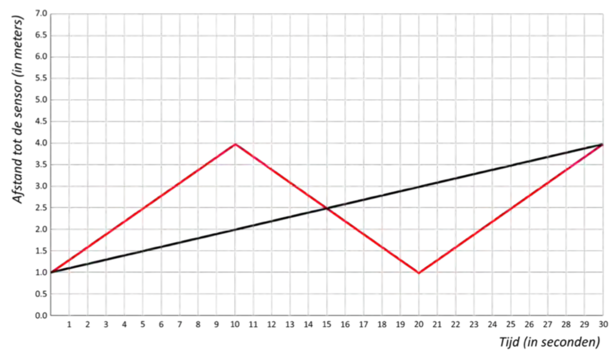


6

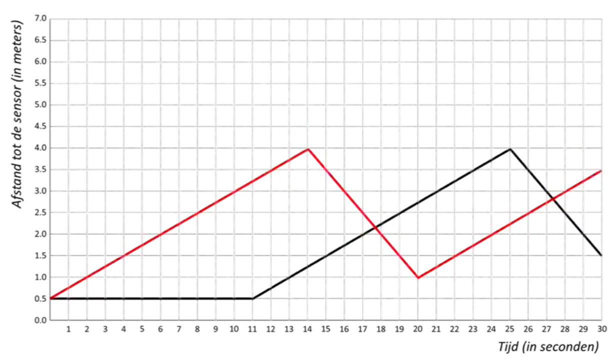
Multiple movements and their graphical representation



Ruimte voor je beschrijving:

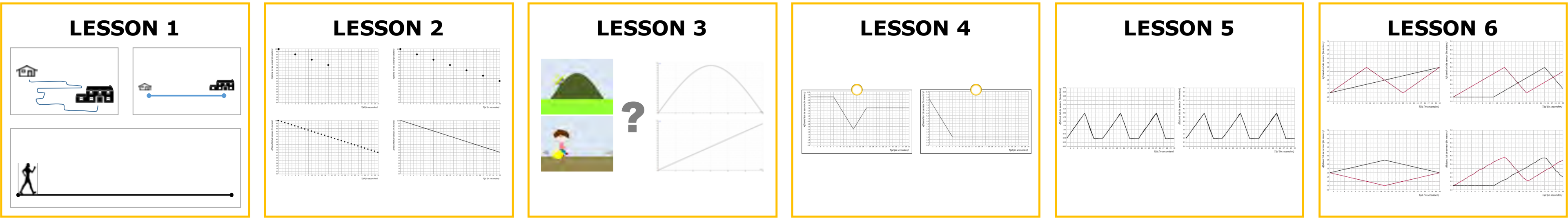


Ruimte voor je beschrijving:



Ruimte voor je beschrijving:

Key components of the Flatland teaching sequence for DYNAMIC DATA MODELING



Focus on: *Reasoning about graphical representations of change*

Specifically: *Reasoning about, and interpreting, time-distance-graphs*

Context: *Moving in front of a motion sensor*

Embodiment: *Experience of moving through space – graph (covariation)*

How to teach EARLY PROBABILITY?

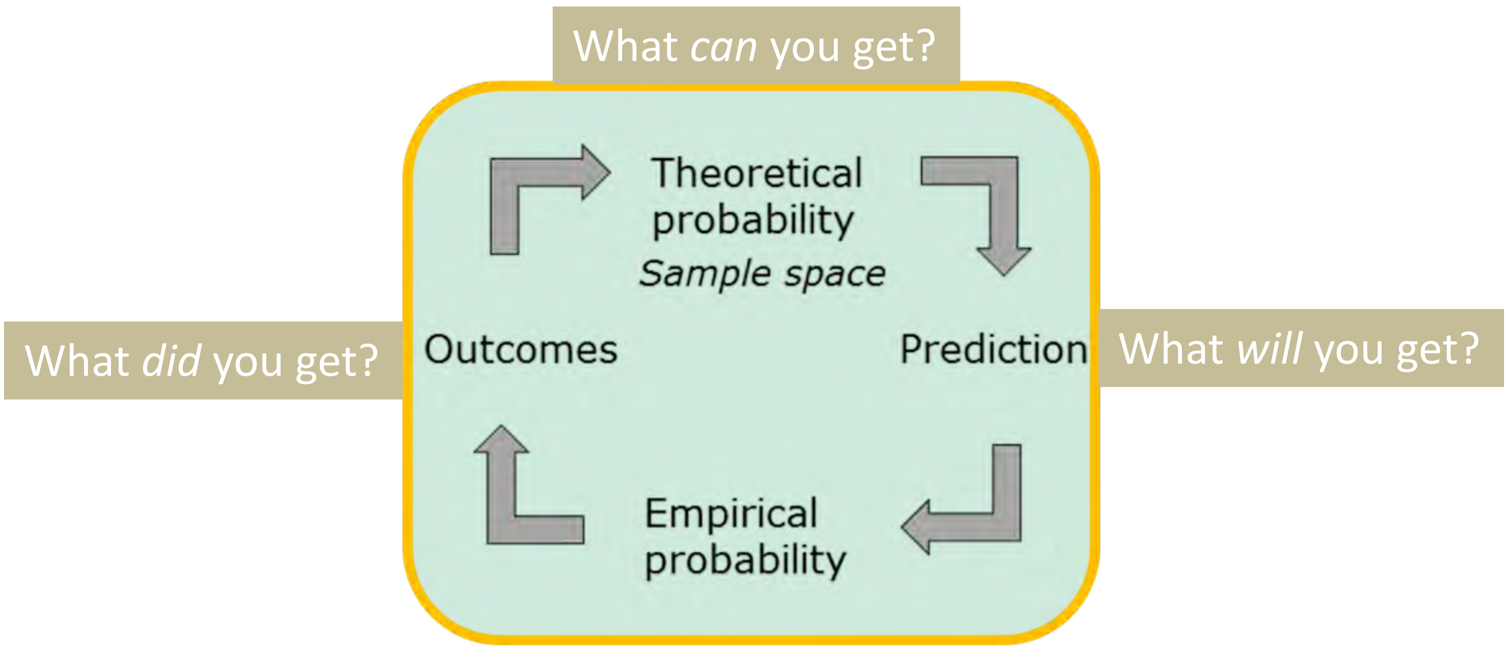
Common approach ↔ *Our approach*

Common approach	*Our approach*
• Doing experiments	• Exploring the **sample space**
• Seeing what comes out	• Predicting what comes out
• Explaining the results	• Doing one experiment
• Exploring the sample size	• Doing many experiments

Four key components of the Flatland teaching sequence for EARLY PROBABILITY

1. Using sample space as a starting point for probabilistic reasoning

2. Three guiding questions
   - What can you get?
   - What will you get?
   - What did you get?



Four key components of the Flatland teaching sequence for EARLY PROBABILITY

1. Using sample space as a starting point for probabilistic reasoning

2. Three guiding questions
   - What can you get?
   - What will you get?
   - What did you get?

3. Supporting perspective switches between
   - Unpredictability <-> Predictability
   - Theoretical probabiliy <-> Empirical probability
   - Elementary results <-> Types of results

4. Experiments with physical chance generators and computer simulations

Structure of the Flatland teaching sequence for EARLY PROBABILITY

1. Exploring basic concept of probability
2. Exploring throwing with one die: one throw, many throws

1 chance generator

3. Exploring two coins: elementary results and types of results
4. Exploring two dice: elementary results and types of results

2 chance generators

5. Exploring the marble scooper

4 chance generators

one experiment

and

many experiments

6. Discovering similarities between different chance situations

➢ Sorting events

This will **certainly** happen	This will **maybe** happen	This will **certainly not** happen

The train from Utrecht to Amsterdam has tomorrow a delay of an hour

If you always water a plant it will stay alive

… week everything is for free in the supermarket

If you never water a plant it will die

Taking a blue marble from a bag containing nine blue and one red marble

If you enter a lottery you win a prize

1

Exploring basic concept of probability

➢What can you throw? What will you throw?

Regular die

1 What numbers can you throw? {1,2,3,4,5,6} →*Theoretical probability (sample space)*

2 What number will you throw? → *Notion of chance*

Adapted die

3 What numbers can you throw? {1,2,3,4,5,5} →*Theoretical probability (sample space)*

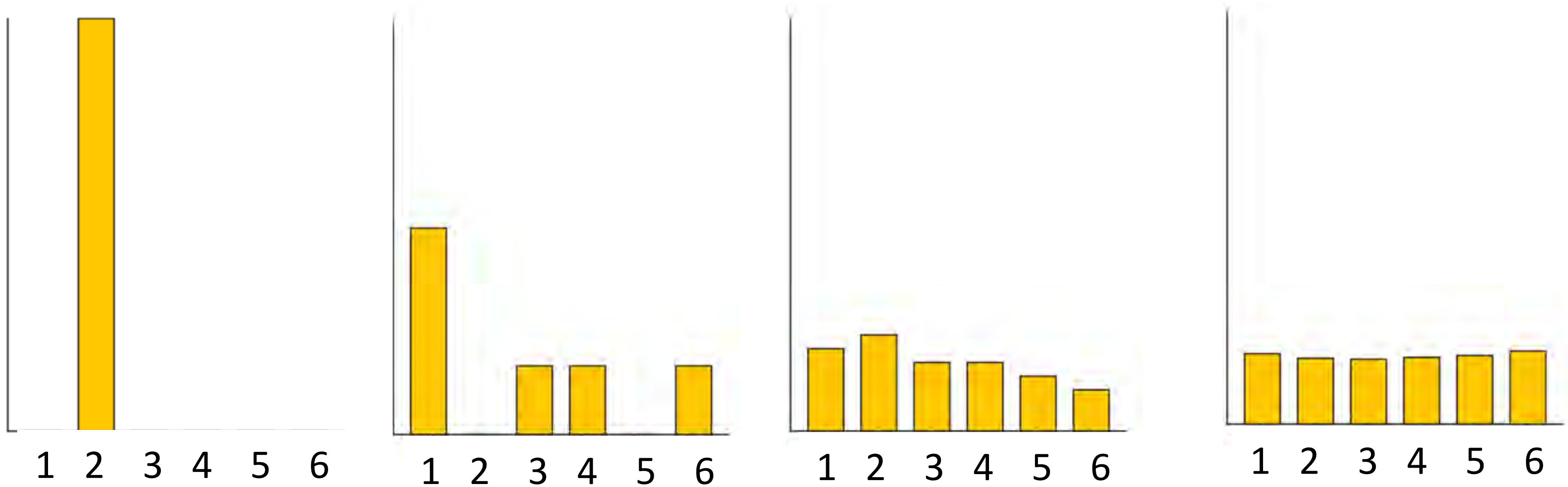
4 What number will you throw? → *Notion of chance*

5 Think of a die with which it would be easier for you to predict what you will throw? →***Manipulating** the theoretical probability*

Exploring throwing with one die: one throw, many throws

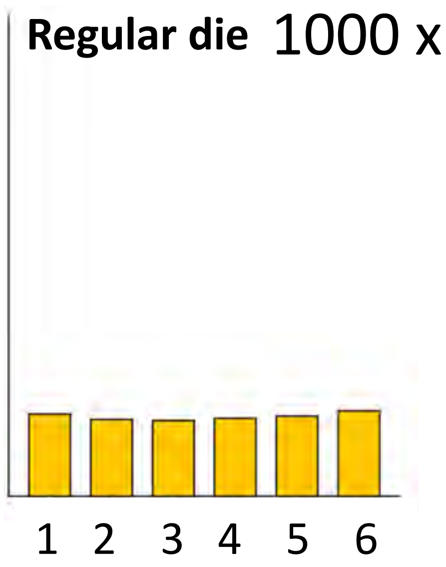
➢Predicting, throwing, and looking what you get

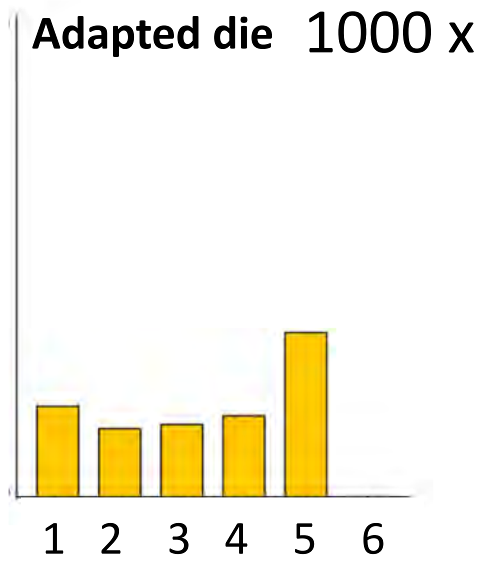
Regular die 1 x 6 x 30 x 1000 x



Exploring throwing with one die: one throw, many throws

➢Predicting, throwing, and looking what you get





What **result** will you get?

→Connecting theoretical probability to empirical probability

➢Look and predict

What kind of **die** is used?

→Connecting empirical probability to theoretical probability

Exploring two coins: elementary results and types of results

➢ Tossing two coins once

What can you get?

1st coin	2nd coin	3 types of results
H	H	→ 2xH
T	T	→ 2xT
H	T	→ H+T
T	H	

➢ Tossing two coins many times

The coins are tossed 100 times. What do you think, who will win?

Tim chooses 2xH

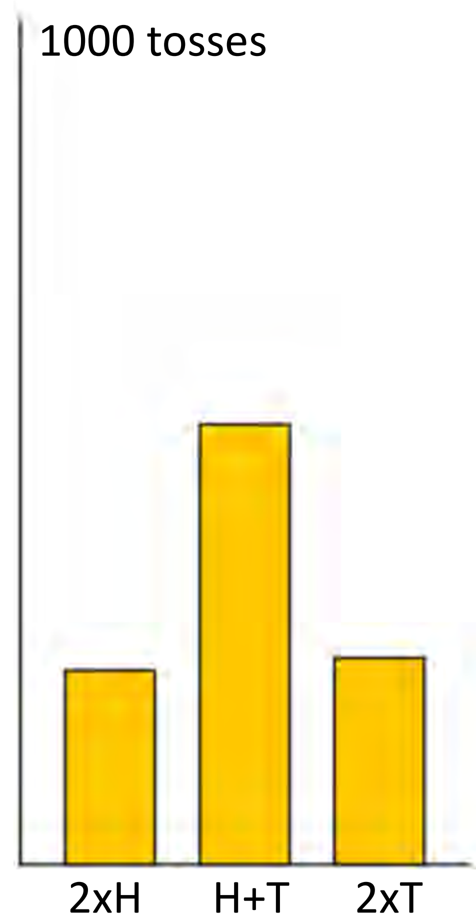
Lisa chooses H+T

Richard chooses 2xT

➢ Tossing two coins many times, trying it out

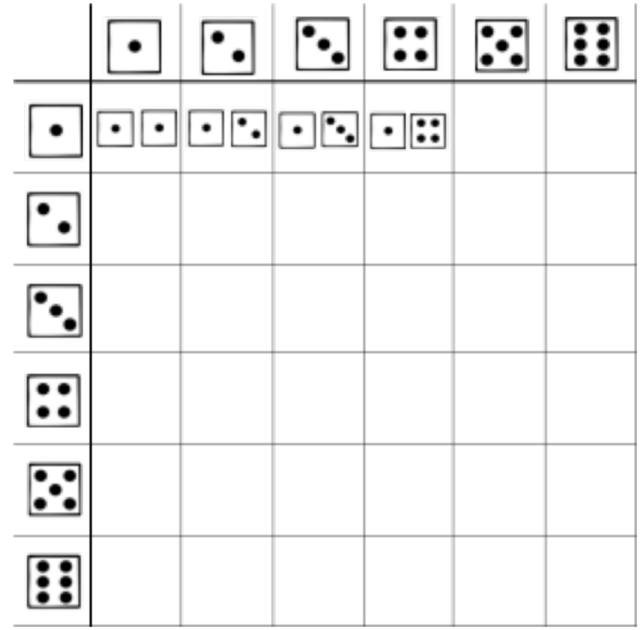
Tossing and stacking pieces of wood

Simulating on the computer



4

Exploring two dice: elementary results and types of results

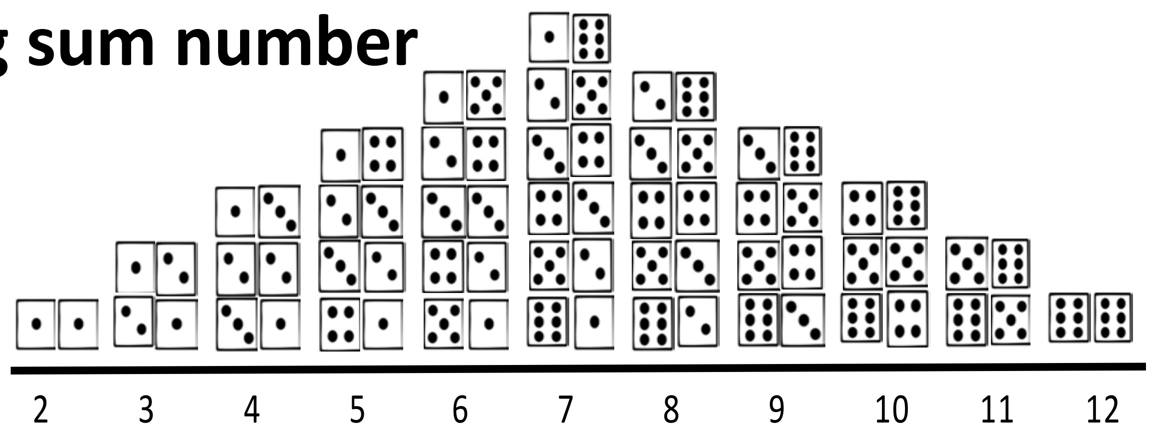


➢ Throwing two dice once

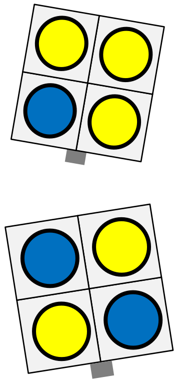
What results can you get?
How many combinations are possible?
What sums can you get?

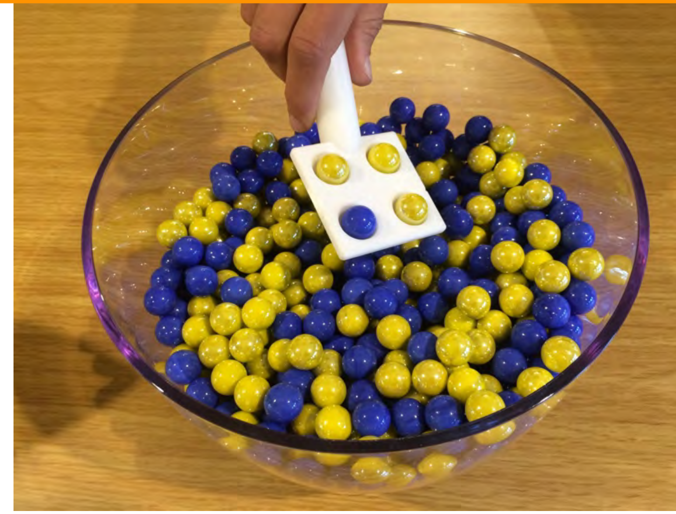
➢ Throwing two dice many times and predicting the winning sum number

Combination tower



Exploring the marble scooper *idea from Dor Abrahamson*

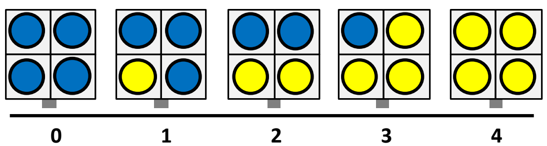




➢Scooping once

What can you scoope?
→All elementary results

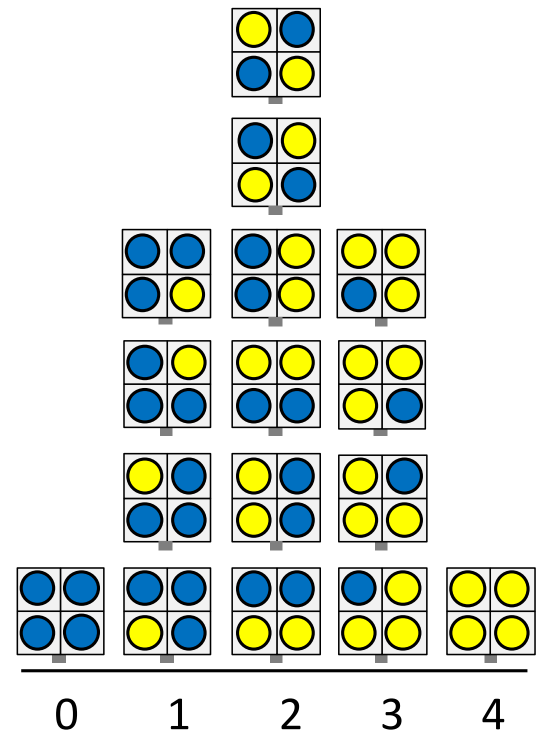
What numbers of yellow marbles can you scope?
→All results of a particular type



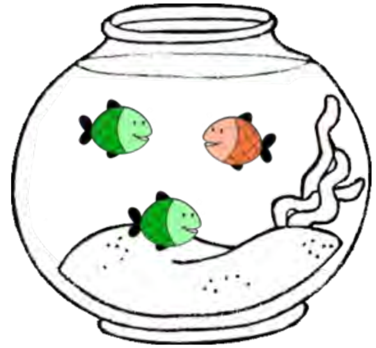
➢Scooping many times

Gambling with the marble scooper.
What number of yellow marbles will be the winning number?

Combination tower



Discovering similarities between different chance situations



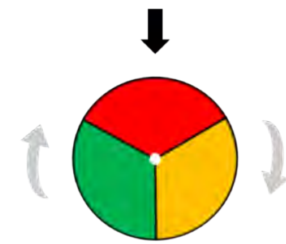
➢Which of these situations match the above?

Throwing a 5 or a 6 with a die.
YES/NO

Why?

Throwing two heads with two coins.
YES/NO

Why?



Getting red when turning the spinner.
YES/NO

Why?