Freedom of Design: The Multiple Faces of Subtraction in Dutch Primary School Textbooks

Marc van Zanten and Marja van den Heuvel-Panhuizen

Abstract Mathematics textbook series largely determine what teachers teach and consequently, what students learn. In the Netherlands, publishers have hardly any restrictions in developing and publishing textbooks. The Dutch government only prescribes the content to be taught very broadly and does not provide guidelines on how content has to be taught. In this study, the consequences of this freedom of design are investigated by carrying out a textbook analysis on the topic of subtraction up to 100. To examine the relationship between the intended curriculum and the potentially implemented curriculum, we analyzed the mathematical content and performance expectations of two Dutch textbook series. In order to get a closer view of the learning opportunities offered, the learning facilitators of the textbook series were also analyzed. The results of the analysis show that the investigated textbook series vary in their agreement with the intended curriculum with respect to content and performance expectations. The textbook series reflect divergent views on subtraction up to 100 as a mathematical topic. Furthermore, they differ in the incorporated ideas about mathematics education, as shown in the learning facilitators they provide. Consequently, the examined textbook series provide very different opportunities to students to learn subtraction up to 100.

Keywords Textbook analysis · Subtraction up to 100 · Mathematical content · Performance expectations · Learning facilitators · Intended curriculum · Potentially implemented curriculum

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Introduction

Textbooks are of great importance in mathematics education. They mediate between the intended curriculum (the statutory goals of education) and the implemented curriculum (the actual teaching in classrooms). Therefore, textbooks are referred to as the potentially implemented curriculum (Valverde et al. 2002). Mathematics textbook series largely determine what teachers teach and, consequently, what students learn (Stein and Smith 2010). Although teachers' teaching is not always in alignment with the textbook they use (Weiss et al. 2002), the textbook is for many teachers the decisive source to realize their mathematics teaching. In the Netherlands, textbooks have a determining role in daily teaching practice. In recent studies it was found that 94 % of the teachers indicate that a textbook is the main source of their teaching (Meelissen et al. 2012) and at least 80 % of primary school teachers are following more than 90 % of the textbook content (Hop 2012).

The intended curriculum and what shows up in a textbook series is not always the same. Textbooks are not only influenced by educational goals, but also by other factors such as commercial considerations, concerns about underprepared teachers (Weiss et al. 2002) and the existence of different ideas about the nature of mathematics that should be emphasized, as well as what instructional approaches should be applied (Reys and Reys 2006). Differences may appear during the transition from the intended curriculum to the potentially implemented curriculum, particularly in countries where there is no centralized textbook design.

In the Netherlands, there is no authority which recommends, certifies or approves textbook series before they are put on the market. Thus, publishers have hardly any restrictions in developing and designing textbook series. In order to investigate the consequences of this freedom of design, we examined in two textbook series how the Dutch intended curriculum is 'translated' into content in the form of tasks, performance expectations, and learning facilitators. To unambiguously determine the possible consequences of this freedom of design, we chose an apparently simple and straightforward mathematical topic for our analysis: subtraction up to 100.

Context and Focus of the Study

Textbook Development in the Netherlands

Freedom of educational design in a way follows from the Dutch constitutional 'freedom of education'. Originating from an arrangement that gave parents the right to found schools in accordance with their religious views, freedom of education has been laid down in the Constitution since 1917. Nowadays, it also allows schools to be founded based on particular pedagogical and instructional approaches.

Because of the freedom of education, the government is rather restrained in giving instructional prescriptions. This means that the Ministry of Education prescribes only the 'what', the subject matter content to be taught, and not the 'how', the way in which this content is to be taught. Not having guidelines for the 'how' gives textbook authors the opportunity to bring in their own views and ideas on teaching mathematics.

There is another reason why textbook authors can express their own interpretations. For several years the 'what' in the intended mathematics curriculum was only described very broadly in the Core Goals for primary school (OCW 1993, 2006). It was not until 2009 that the Core Goals were extended with the Reference Standards (OCW 2009), describing in more detail what students should be able to at the end of primary school. However, there is still room for interpretation. For example, the Reference Standards state that students should learn to calculate using a standard method, but they do not prescribe what standard method should be taught.

There are ten textbook series[1] for teaching primary school mathematics on the market in the Netherlands. The newest have all been released between 2009 and 2012. Several have a history of earlier editions, including two that date back to the 1970s and 1980s,[2] when a reform movement in mathematics education was being enacted in the Netherlands. This reform movement was aimed at developing an alternative for the then prevailing mathematics education, which had a very mechanistic character, and in which teaching began at a formal, symbolic level. To give children a better basis for understanding mathematics, Freudenthal and the Wiskobas group developed a new approach to mathematics education in which, among other things, the use of contexts to encourage insight and understanding played a crucial role. This reform, which was later called 'Realistic Mathematics Education' (RME) (e.g., Van den Heuvel-Panhuizen 2001), was largely supported by reform-oriented textbook series.[3] Until recently all Dutch textbooks series were based more or less on this approach to teaching mathematics and they were all labeled by their publishers as 'realistic'. However, due to a debate that has taken place in the Netherlands since 2007 criticizing the RME approach in favor of a return to the traditional, mechanistic approach (Van den Heuvel-Panhuizen 2010) some textbook series have adapted their content (more emphasis on algorithms[4]) and teaching approach (more attention to repetition[5]) in their new editions. Moreover, new textbook series have been released that are presented as an alternative for realistic textbook series, that restore the traditional mechanistic approach with only one calculation method for each

[1] 'De Wereld in Getallen', 'Pluspunt', 'Rekenrijk', 'Alles Telt', 'Talrijk', 'Wis en Reken', 'Wizwijs', 'Reken Zeker', 'Rekenwonders' en 'Het Grote Rekenboek'.

[2] 'De Wereld in Getallen' developed from 1975 on, and Pluspunt, the development of which started in 1985.

[3] This underlines the crucial role that mathematics textbooks have in the Netherlands.

[4] A folder released for the textbook series 'De Wereld in Getallen' (4th edition) and 'Pluspunt' (3rd edition) says "Algorithms get more attention and are gradually built up until the classic long division appears." (All translations of folders and examples from textbooks are done by the authors of this chapter.)

[5] A folder released for the textbook series 'De Wereld in Getallen' (4th edition) and 'Pluspunt' (3rd edition) says: "There is much more room for practice, repetition and automatization."

operation and a step-by-step approach with a focus on repetition.[6] Furthermore, a new textbook series which is a Dutch version of a textbook series developed in Singapore[7] was published. Thus, as a result of the debate about mathematics education, the corpus of Dutch mathematics textbooks series has become very diverse.

Subtraction in the Dutch Intended Curriculum

According to the current Dutch Core Goals for primary school mathematics, children have to "learn to use mathematical language and have to gain numeracy and mathematical literacy" (OCW 2006, p. 37). Mathematical language includes arithmetical and mathematical terms and notations. Mathematical literacy and numeracy refer to, among other things, coherent insight in numbers and a repertoire of number facts and calculation methods. Furthermore, the Core Goals indicate that children "learn to ask mathematical questions and formulate and solve mathematical problems [. . .] and explain the solutions in mathematical language to others" (OCW 2006, p. 39). Concerning the basic operations, the Core Goals mention that students learn to calculate both in smart ways and using standard methods (OCW 2006, p. 43). Specifically concerning subtraction up to 100, the Core Goals state that children "learn to quickly carry out the basic calculations in their heads using whole numbers, at least up to 100, with additions and subtractions up to 20[. . .] known by heart" (OCW 2006, p. 43).

The Dutch Reference Standards for mathematics (OCW 2009) distinguish three types of knowing: 'knowing-what', 'knowing-how' and 'knowing-why'. With this in mind, the Standards can be considered a description of what Valverde et al. (2002, p. 125) call "expectations of performance" which refers to "what students should be able to do with content." 'Knowing-what' relates to knowledge of number facts and calculation methods. Subtraction up to 100 includes mental calculation, both using standard methods and using properties of numbers and operations. Furthermore, students learn to subtract both by taking away and by determining the difference. 'Knowing-how' refers to making functional use of particular number facts and calculation methods, including using standard methods with insight in real-life situations and converting context situations to bare number problems. 'Knowing-why' refers to understanding. This includes, for example, knowledge about the operations, such as knowing that the commutative property does not apply to subtraction as it does to addition.

[6] A folder released for the textbook series 'Reken Zeker' says: "Practice, practice and more practice", "One strategy for all children". A folder released for the materials of 'Het Grote Rekenboek' says: "This textbook series gives an answer to the recent criticism on mathematics education."

[7] A folder released for the textbook series 'Rekenwonders' says: "This is the Dutch edition of an extremely successful and internationally praised Singaporean textbook."

A Mathedidactical Analysis of Subtraction up to 100

Subtraction as a Mathematical Concept

Relationships between whole numbers can be additive and multiplicative. These relationships ensure that one can think of and reason within an interrelated number system instead of having to deal with an innumerous set of individual loose numbers (Kilpatrick et al. 2001). The additive and multiplicative relationships interconnect, combine, and generate numbers.

Addition and subtraction refer to additive number relationships. This implies that the numbers involved reflect a part-whole relationship. Combining parts into a whole can be considered an addition, whereas taking a part from a whole can be considered a subtraction. Furthermore, the operation of subtraction is the inverse of addition: subtraction undoes addition and vice versa (if $a + b = c$, then $c - b = a$).

Although subtraction is mostly associated with removing a part from a whole, it has two phenomenological appearances: taking away and determining the difference (Van den Heuvel-Panhuizen and Treffers 2009). The two manifestations of subtraction reflect two meanings of subtraction. These two different semantic structures can nevertheless be expressed by the same symbolic representation: $c - b = a$. Written as a minuend minus a subtrahend it can literally stand for taking away b from c, but it can also represent comparing c and b to find the difference, for example, by adding on. So, depending on the semantic structure behind the symbolic representation, the answer to a subtraction problem can have two different meanings: a remainder and a difference (Usiskin 2008).

Just like the minus symbol in the symbolic representation $c - b = a$ does not always mean taking away, the operation of subtraction is not exclusively restricted to problems in which the minus symbol appears (Freudenthal 1983). For example, problems with a + symbol in the form of $\cdots + b = c$ and $a + \cdots = c$ can be solved by a subtraction operation. These latter problems are actually subtraction problems in an addition format (Selter et al. 2012).

Calculation Methods for Subtraction up to 100

The methods that can be applied for carrying out subtractions up to 100 can be described from both the number perspective and the operation perspective (Van den Heuvel-Panhuizen 2012; Peltenburg et al. 2012) (see Fig. 1).

From the operation perspective, subtraction problems up to 100 can be solved by (1) taking the subtrahend away from the minuend, (2) adding on from the subtrahend until the minuend is reached, and (3) taking away from the minuend until the subtrahend is reached. These procedures are respectively called: direct subtraction (DS), indirect addition (IA), and indirect subtraction (IS) (De Corte and Verschaffel 1987; Torbeyns et al. 2009).

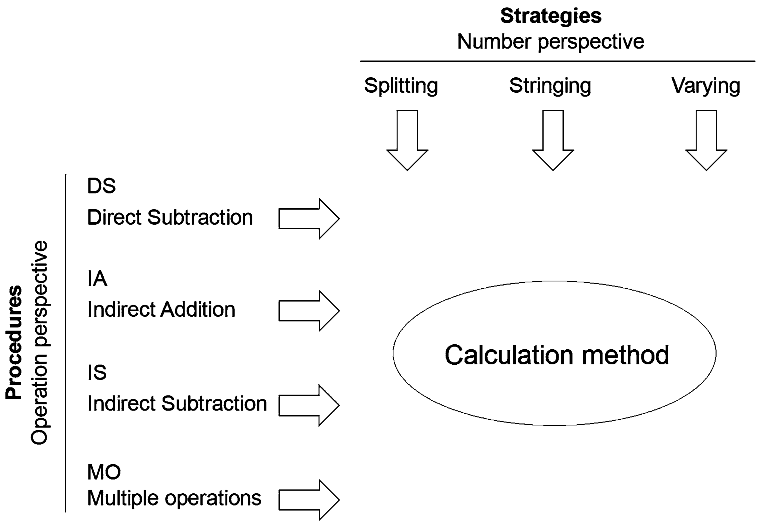

Fig. 1 Two perspectives for describing calculation methods for subtraction up to 100

The number perspective describes how the numbers involved are dealt with. Roughly speaking, there are three strategies: splitting, stringing, and varying. Although researchers do not always use the same wording—for example, other expressions can be found in Klein et al. (1998) and Torbeyns et al. (2009)—there is broad agreement about the general meaning of these strategies. In the splitting strategy, the minuend and the subtrahend are split into tens and ones and then the tens and ones are processed separately. In the stringing strategy, the minuend is kept intact and the subtrahend is decomposed in suitable parts which are subtracted one after another from the minuend. When a varying strategy is applied, the minuend and/or the subtrahend are changed to get an easier subtraction problem. Although in theory all three strategies can be combined with each of the four procedures, not all combinations are common or suitable (see for a more detailed discussion, Peltenburg et al. 2012).

DS can be applied with both splitting (e.g. $67 - 41$ is solved by $60 - 40 = 20$ and $7 - 1 = 6$, followed by $20 + 6 = 26$) and stringing (e.g. $67 - 41$ is solved by $67 - 40 = 27$ and $27 - 1 = 26$). Both the IA and IS procedures can also be combined with splitting and stringing. For example, in the case of $67 - 41$, applying IA with a splitting strategy means calculating $40 + 20 = 60$ and $1 + 6 = 7$, and then $20 + 6 = 26$. Combining IA with a stringing strategy means calculation is $41 + 9 = 50$ and $50 + 10 = 60$ and $60 + 7 = 67$, followed by $9 + 10 + 7 = 26$. Although this latter method can require more steps (when there is a large difference between minuend and subtrahend), the advantage of the stringing strategy is that the problem is not split into two problems. The starting number is kept as a whole.

For subtraction problems that require crossing the ten, applying a DS procedure combined with splitting easily leads to the mistake of reversing the ones (e.g., in the case of $75 - 38$, $70 - 30$ is frequently incorrectly followed by $8 - 5$). This mistake does not happen when DS is combined with stringing. Even more convenient is applying an IA or IS procedure combined with stringing, for example, when there is a small difference between the minuend and the subtrahend, such as in the case of $62-58$. Solving these problems by a stringing strategy combined with IA ($58+2=60$ and $60+2=62$, followed by $2+2=4$) or with the less common IS procedure ($62-2=60$ and $60-2=58$, followed again by $2+2=4$) are easier methods that are less sensitive to errors.

Finally, the varying strategy implies multiple operations. Applying this strategy means that a problem is solved through changing it into another problem by making use of properties of numbers and operations. For example, a problem like $77-29$ can be solved by first calculating $77-30$, followed by $47+1=48$.

Learning Facilitators for Subtraction up to 100

According to Kilpatrick et al. (2001), mathematical proficiency involves five interwoven and interdependent components, including conceptual understanding; procedural fluency; formulating, representing and solving mathematical problems; having the capacity for reflection and justification; and seeing mathematics as useful and worthwhile. Following this interpretation of mathematical proficiency—which is also reflected in the Dutch intended curriculum—implies that performance expectations should not be restricted to carrying out routine procedures, but also include flexible application of calculation methods, strategy choice, and contextual interpretation of outcomes (Verschaffel et al. 2007).

Applied to the learning of subtraction up to 100, this means that students should be offered opportunities to build a broad mental constitution of subtraction, including the different semantic structures, symbolic representations, and calculation methods of subtraction. Textbooks can contribute to this broad constitution of subtraction by including didactical support in their exposure to subtraction up to 100, such as sufficient contexts and models.

Contexts First of all, contexts can present students with situations in which subtraction emerges as a mathematical concept in a rather natural manner. The role of contexts is to add meaning to this mathematical concept in order to support the development of understanding. This can happen especially when the contexts that are used are not restricted to word problems in a stereotyped text frame, but instead come in a variety of forms and refer to students' real-life knowledge (De Corte and Verschaffel 1987). Thus, students can become aware that subtraction can apply to all kinds of situations, reflecting different meanings of subtraction. For example, eating cookies and ascertaining how many are left, filling an album with photos and determining how many can still be included, and figuring out how many centimeters a particular person is taller than another person. These contexts which refer to different semantic structures of subtraction can prompt students to use either the DS or the IA or IS procedure.[8] By manipulating the variety in contexts, textbooks can support students' understanding of the different semantic structures of subtraction and learning various calculation methods to solve subtraction problems (see also Fuson 1992). We refer to this use of contexts as ‘contexts for supporting understanding’, which we distinguish from the use of contexts for just applying subtraction methods. The latter reflects a performance expectation rather than a form of didactical support. To make a clear distinction between these two functions of contexts, in this study we interpreted contexts for supporting understanding as contexts that serve as a source for something new to be learned, such as a new calculation method.

Models Besides contexts, models are also important to support students' learning of subtraction up to 100. This is especially true for carrying out calculation methods and specifically applies to the strategies that are used. A requirement for making this support of models effective is that the models that are used match the strategies used (Van den Heuvel-Panhuizen 2008). Models and strategies should be epistemologically consistent. This means that, for example, the splitting strategy and the stringing strategy each have their own supporting models. The splitting strategy, which is strongly related to the cardinal aspect of numbers, can best be supported by a group model that also reflects the cardinal aspect, like base-10 arithmetic blocks. Likewise, the stringing strategy, which is strongly related to the ordinal aspect of number, finds its supportive model equivalent in line models such as a number line. A line model is also suitable for visualizing and supporting a varying strategy. For example, in the case of $78 - 29$ this means first making a backward jump of 30, followed by a forward jump of 1. As stated earlier, solving $78 - 29$ by a splitting strategy easily leads to the mistake of reversing the ones. A line model would not help to overcome this difficulty, because dealing separately with the 70 and the 20, and the 8 and the 9 on a number line does not make sense. In other words, in teaching calculation methods, strategies and models should match, otherwise models do not have the supportive function they are assumed to have. Consequently, depending on the strategy that is intended, textbooks should give more attention either to group models or to line models.

Symbolic Representations Building a broad mental constitution of subtraction also requires that students are offered various symbolic representations of subtraction. Besides the standard representation $c - b = \cdots$, students should also have opportunities to deal with alternative symbolic representations such as $c - \cdots = a$ and $a + \cdots = c$. These problems make it clear that the operation symbol in a problem can have different meanings (Fuson 1992), and is not per se equivalent to the operation that can be applied to find the solution of that problem. The different symbolic representations reflect the part-whole aspect of additive number relationships

[8] This use of contexts should fade away after some time. After all, even though a context can steer a certain calculation method, in term, in the decision what calculation method will be used, not the context, but the numbers involved play a key role.

and the link between addition and subtraction. Furthermore, it supports the understanding that the = symbol does not only mean 'results in' but also 'is equivalent to'. According to Fuson (1992), textbooks do not always pay much attention to the different meanings of the equal and operation symbols.

Research Questions

The purpose of this study is to reveal the consequences of freedom of design for Dutch textbooks as the potentially implemented curriculum for primary school and for the learning opportunities that students are offered. Focusing on subtraction up to 100, we came up with the following research questions:

1. Do Dutch mathematics textbooks reflect the *content* of the Dutch intended curriculum concerning subtraction up to 100?
2. Do Dutch mathematics textbooks reflect the *performance expectations* of the Dutch intended curriculum concerning subtraction up to 100?
3. What *learning facilitators* for learning subtraction up to 100 are incorporated in Dutch mathematics textbooks?

Method

To answer the research questions, a textbook analysis was carried out in which we examined two Dutch textbooks series. The analysis focused on three perspectives: the mathematical content, the performance expectations and the learning facilitators.

Textbook Materials Included in the Analysis

To include the full scope of didactical approaches in the Netherlands in our analysis we examined two recently developed textbook series that, although from the same publisher, are clearly positioned in two contrasting approaches to mathematics education (see section "Textbook Development in the Netherlands"). The first textbook series, called 'Rekenrijk' (RR) (Bokhove et al. 2009), is a RME-oriented textbook series. The name 'Rekenrijk' means both 'kingdom of arithmetic' and 'rich arithmetic'. The second textbook series, called 'Reken Zeker' (RZ) (Terpstra and De Vries 2010), is a new textbook series that is presented as an alternative for realistic textbook series. The name of this textbook series means 'arithmetic with certainty'.

Because subtraction up to one hundred is mainly taught in grade 2, the textbook analysis was carried out with textbook materials from this grade only. We analyzed all materials for grade 2 that are meant for all students. Textbook materials meant for evaluation, and subsequent optional lessons for repetition or enrichment, were not included in our analysis.

Perspective	Category	Subcategory	
Content	Types of problems	Prerequisite knowledge	Decomposing numbers up to 10
			Backwards counting with tens
		Subtraction up to 10	
		Subtraction up to 20	Without bridging the ten
			Bridging the ten
		Subtraction up to 100	Without bridging a ten
			Bridging a ten
	Format of problems	Bare number problems	
		Context problems	
	Semantic structure of problems	Subtraction as taking away	
		Subtraction as determining the difference	
Performance expectations	Knowing subtraction facts	Knowing subtraction facts up to 10	
		Knowing subtraction facts up to 20	
	Carrying out subtractions	Using standard methods	
		Using alternative methods	
	Applying subtractions	Using subtraction methods in context problems	
	Understanding subtraction	Giving explanations	
		Choosing an appropriate method	
Learning facilitators	Degree of exposure	Number and distribution of tasks	
	Structure of exposure	Sequence in types of problems	
		Sequence in level of abstraction	
	Didactical support in exposure	Use of contexts for supporting understanding	
		Use of models	
		Use of various symbolic representations	
		Use of textual instructions	

Fig. 2 Framework for textbook analysis

Framework for Textbook Analysis

To analyze the textbook materials we developed a framework containing the perspectives of content, performance expectations, and learning facilitators (see Fig. 2). Most categories within these three perspectives were initially formulated on the basis of the Dutch intended curriculum for subtraction (see section "Subtraction in the Dutch Intended Curriculum") and our mathedidactical analysis of subtraction up to 100 (see section "A Mathedidactical Analysis of Subtraction up to 100"). Several subcategories were established after an initial round of the analysis, based on what we actually found in the textbook series.

Content

The perspective of content involves problem types, problem formats, and semantic structures of the problems presented in the textbook materials. Regarding the problem types we made a subdivision based on the number domain involved. We incorporated relevant prerequisite knowledge for subtraction: decomposing numbers up to 10 and counting backwards with tens. For the format of the problems we made a distinction between bare number problems and context problems. The semantic structure of problems refers to the two phenomenological appearances of subtraction.

Performance Expectations

Regarding performance expectations, we included knowing subtraction facts, carrying out subtractions, applying subtractions and understanding subtraction. The first two categories correspond to 'knowing-what', the third to 'knowing-how' and the fourth to 'knowing-why', as described in the Dutch Reference Standards. Knowing subtraction facts is subdivided into knowing subtraction facts up to 10 and knowing subtraction facts up to 20. Carrying out subtractions is subdivided into using standard calculation methods (DS combined with splitting or stringing) and alternative calculation methods (e.g., IA combined with stringing or MO combined with a varying strategy). This distinction is in agreement with the Dutch intended curriculum. Applying subtractions refers to using already learnt subtraction facts and calculation methods in context problems. For the category 'understanding' we distinguished 'giving explanations' and 'choosing an appropriate method', based on performance expectations found in the first round of analysis, that go beyond knowing, carrying out and applying subtractions, and unambiguous apply to understanding.

Learning Facilitators

With respect to learning facilitators, we included degree and structure of exposure, based on the importance of the amount and sequencing of content in textbooks (Valverde et al. 2002). We included didactical support in exposure based on our mathedidactical analysis. The subcategory 'use of textual instructions' was added after the first round of the textbook analysis, again based on what we found in the textbook series that can also be considered as supporting learning.

Unit of Analysis

In both textbooks series, the content is organized in lessons meant for one mathematics hour. These lessons are subdivided into sets of tasks. In our study, we use the term 'task' to refer to the smallest unit that requires an answer from a student. Because the amount of tasks vary per set of tasks (see Fig. 3), and content and performance expectations may vary per single task, we used the task as unit of analysis.

Analysis Procedure

First, we identified all subtraction-related tasks. After an initial round of analysis was carried out, we added the following subcategories: 'giving explanations', 'choosing an appropriate method' and 'use of textual instructions'. Then, the first

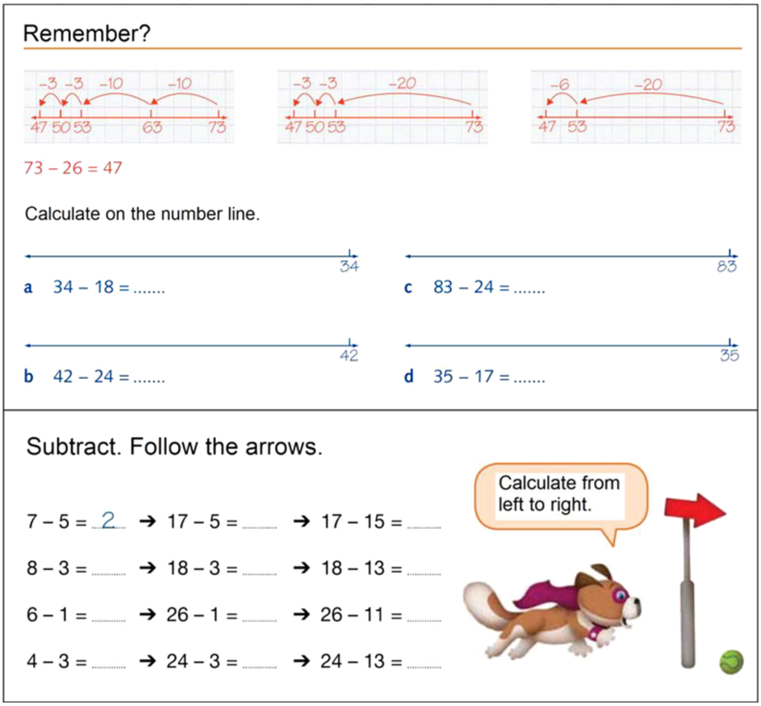

Fig. 3 RR set of 4 tasks (above, RR-book 4b-1, p. 30) and RZ set of 11 tasks (below, RZ-book 4c, p. 26). In the Netherlands, K1, K2, grade 1 and grade 2 are respectively called group 1, 2, 3 and 4

author of this chapter coded all subtraction-related tasks according to the final version of our framework. Each task received several codes. For the content, a code was given for the problem type, the problem format, and the semantic structure of the problem. For the performance expectations, each task was first coded as knowing subtractions facts, carrying out subtractions or applying subtractions. If neither of these sub-categories was applicable, no code was given. Next, for each task, if applicable, a code was given for the category understanding of subtractions. For the learning facilitators, the degree of exposure was determined from the number of tasks. Because the tasks were counted in consecutive lessons, we got an overview of the distribution of the subtraction-related tasks. This also made it possible to reveal the structure of exposure, i.e., the sequence in types of tasks and in level of abstraction. Finally, for each task it was checked which subcategories of didactical support were applicable.

A reliability check of the coding was based on an independent coding by two teacher-trainees. To that end we used a selection of about one tenth of all subtraction-related tasks in which all categories of the framework were included. The two teacher-trainees reached a 93 % agreement. The agreements between each of the teacher-trainees and the first author were respectively 93 % and 95 %.

Results

Content

A substantial difference between the two textbook series for grade 2 is the number of tasks included. The total number of tasks in RR is 5331, whereas RZ has 7051 tasks. However, of these amounts of tasks the proportion of subtraction-related tasks is about the same in both textbooks: RR contains 22 % subtraction tasks (1166 tasks) and RZ 20 % (1440 tasks).

Types of Problems

Both grade 2 textbook series concentrate more on tasks involving subtraction between 20 and 100, and less on tasks involving subtraction up to 10 and up to 20 (see Table 1). Regarding subtraction up to 20, RR offers more tasks that require bridging the ten than RZ. Within subtraction tasks up to 100, the number of tasks that require bridging a ten is larger in RZ, but relatively RR offers more tasks concerning this type of problem (in RR: 378 out of 572 tasks, is about 66 %; and in RZ: 480 out of 1096 tasks, is about 44 %).

The amount of attention to the prerequisite knowledge for these problems differs. Regarding decomposing numbers up to 10, RR has a substantial number of such tasks and RZ almost none. For counting backwards with tens (e.g., 46; 36; 26), RR has very few tasks, while RZ has none. When we checked whether, for example, decomposing numbers up to 10 is already dealt with in grade 1, we found that both textbook series did indeed put more of an emphasis on this prerequisite knowledge in grade 1 than in grade 2. However, the RR booklets for grade 1 have 418 such tasks, while RZ offers only 167 in its first-grade booklets. So, with respect to providing prerequisite knowledge for subtraction up to 100, there is a large difference between the two textbooks series.

Format of Problems

Both textbook series contain far more bare number problems than context problems (see Table 2). However, RR encloses much more context problems than RZ, both relatively and absolutely, even though in RZ the total number of subtraction tasks is larger than in RR.

Semantic Structure of Problems

In both textbook series, only a minority of the tasks reflect a clearly distinguishable semantic structure. Both textbook series address subtraction as taking away, but subtraction as determining the difference is only dealt with in RR (see Table 3).

Table 1 Types of problems in subtraction-related tasks in RR and RZ in grade 2[a]

Types of problems	RR-tasks		RZ-tasks	
	f	%	*f*	%
Prerequisite knowledge	130	11 %	5	0 %
Decomposing numbers up to 10	107	9 %	4	0 %
Backwards counting with tens	23	2 %	1	0 %
Subtraction up to 10	153	13 %	78	5 %
Subtraction up to 20	311	27 %	261	18 %
Without bridging the ten	79	7 %	135	9 %
Bridging the ten	232	20 %	126	9 %
Subtraction up to 100	572	49 %	1096	76 %
Without bridging a ten	194	17 %	616	43 %
Bridging a ten	378	32 %	480	33 %
Total number of subtraction-related tasks	1166	100 %	1440	100 %

[a]Some percentages do not seem to add up to 100. This is due to rounding off

Table 2 Format of problems in subtraction-related tasks in RR and RZ in grade 2

Format of problems	RR-tasks		RZ-tasks	
	f	%	*f*	%
Bare number problems	1026	88 %	1415	98 %
Context problems	140	12 %	25	2 %
Total number of subtraction-related tasks	1166	100 %	1440	100 %

Table 3 Semantic structure of problems in subtraction-related tasks in RR and RZ in grade 2

Semantic structure	RR-tasks		RZ-tasks	
	f	%	*f*	%
Taking away	210	18 %	403	28 %
Determining the difference	53	5 %	0	0 %
Both taking away and determining the difference	28	2 %	0	0 %
No distinguishable semantic structure	874	75 %	1037	72 %
Total number of subtraction-related tasks	1166	100 %	1440	100 %

Performance Expectations

Both textbook series contain tasks that clearly focus on certain performances. RR contains 1081 and RZ contains 800 clearly distinguishable performance expecta-

Table 4 Performance expectations reflected in subtraction-related tasks in RR and RZ in grade 2[a]

Performance expectations	RR-tasks		RZ-tasks	
	f	%	*f*	%
Knowing subtraction facts	346	32 %	229	29 %
Knowing subtraction facts up to 10	258	24 %	55	7 %
Knowing subtraction facts up to 20	88	8 %	174	22 %
Carrying out subtractions	513	47 %	546	68 %
Using standard methods	413	38 %	546	68 %
Using alternative methods	100	9 %	0	0 %
Applying subtractions	111	10 %	25	3 %
Understanding subtraction	111	10 %	0	0 %
Choosing an appropriate method	74	7 %	0	0 %
Giving explanations	37	3 %	0	0 %
Total number of performance expectations	1081	100 %	800	100 %

[a]In some tasks we distinguished two performance expectations (e.g., carrying out a subtraction and explaining the calculation method). See also Table 1 note

tions (see Table 4). In both textbook series, most emphasis lies on performance expectations related to carrying out subtractions, followed by knowing subtraction facts. RR contains more expectations on applying subtractions than RZ. Expectations regarding understanding were only found in RR.

Knowing Subtraction Facts

RR contains more performance expectations for knowing subtraction facts than RZ. In RR, most emphasis is on knowing subtraction facts up to 10. In RZ, most emphasis is on knowing subtraction facts up to 20.

Carrying out Subtractions

Using Standard Methods In both textbook series students are expected to learn one standard method for carrying out subtractions up to 20 and up to 100, namely DS combined with stringing. However, the textbook series differ in the way that students are supposed to notate their calculations. In the case of tasks that involve bridging a ten, both textbooks suggest the notation of in-between steps or in-between answers. In RR this is done by writing down under the subtrahend how it is decomposed or by keeping track of the taken-away steps on an empty number line (see Fig. 4).

In RZ, the students have to notate the first in-between answer directly after the = symbol, which is supposed to be followed by the remaining part that has to be

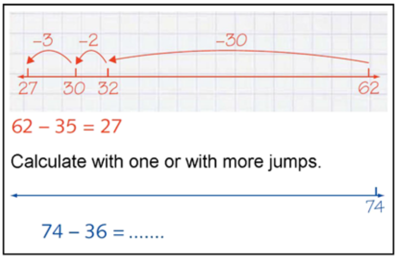

Fig. 4 DS combined with stringing in RR (RR-book 4b-1, p. 57)

taken away (see Fig. 5). Although the symbolic representation that results in the end is mathematically correct (in fact it describes two equivalent subtractions), notating the calculation in this way implies that students have to perform several in-between steps mentally.

Using Alternative Methods Only in RR are students expected to learn alternative subtraction methods also, namely, the procedures IA and IS and a varying strategy (see Fig. 6). Although RZ contains missing number tasks (e.g., $28 - \cdots = 23$) which could prompt IS, this textbook series does not otherwise pay attention to this procedure or to any alternative method.

Applying Subtractions

In both textbook series, contexts are used for the application of calculation methods that are presented earlier. RR offers such contexts more than four times as often as RZ (see Table 4). Both textbook series use contexts that refer to real life situations. In RZ all contexts concern taking-away situations, presented by a series of similar sentences. RR offers contexts referring both to taking away and determining the difference, presented in various ways (see Fig. 7).

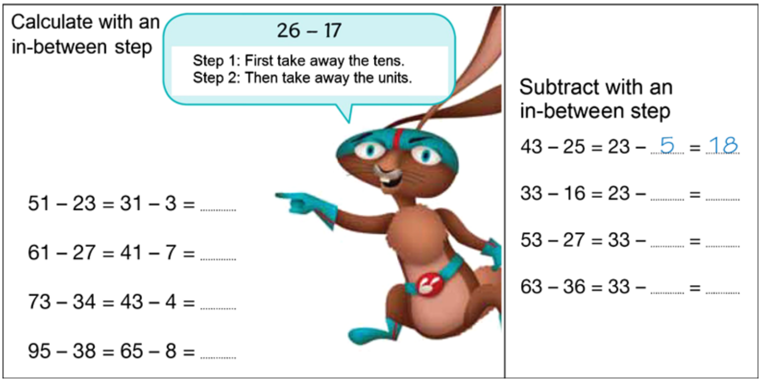

Fig. 5 DS combined with stringing in RZ tasks up to 20 and up to 100 (RZ-book 4c, p. 71; p. 74)

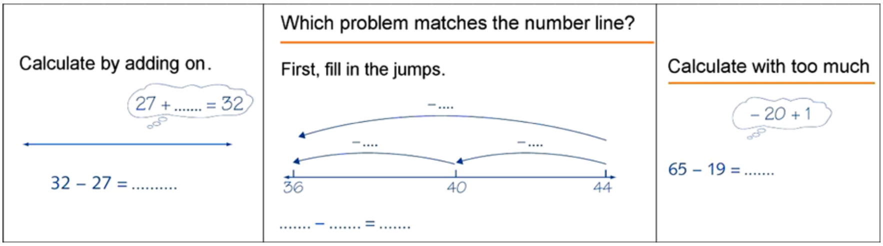

Fig. 6 IA (*left*), IS (*middle*) and a varying strategy (*right*) in RR tasks (RR-book 4b-2, p. 61; 4b-1, p. 2; 4b-2, p. 78)

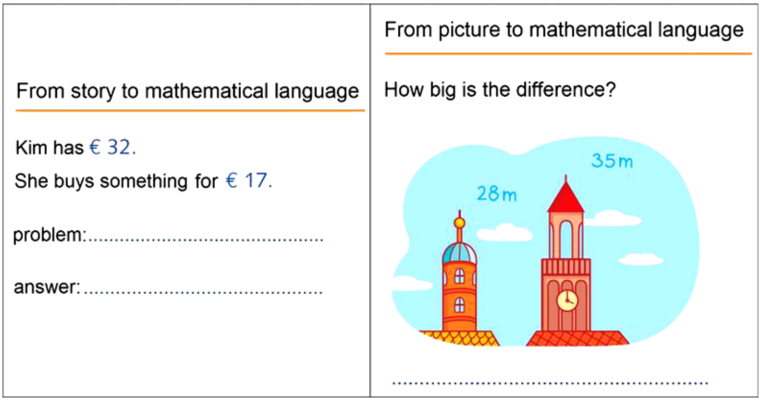

Fig. 7 Context problems in RR reflecting taking away (*left*) and determining the difference (*right*) (RR-book 4b-2, p. 37; p. 78)

Understanding Subtraction

In RR, we found 111 tasks explicitly offering directions or questions to prompt students' reasoning (see Table 4). These tasks include questions for students to explain their thinking (e.g., 'Hoe heb je dit uitgerekend?' [How did you calculate this?]), visualize their calculation method or choose an appropriate calculation method for a given subtraction with certain numbers (see Fig. 8). In RZ, we did not find clearly distinguishable performance expectations regarding understanding.

Learning Facilitators

Degree of Exposure

As mentioned before, RZ provides more subtraction-related tasks (1440) than RR (1166). Figure 9 displays how these tasks are distributed over time (covering the

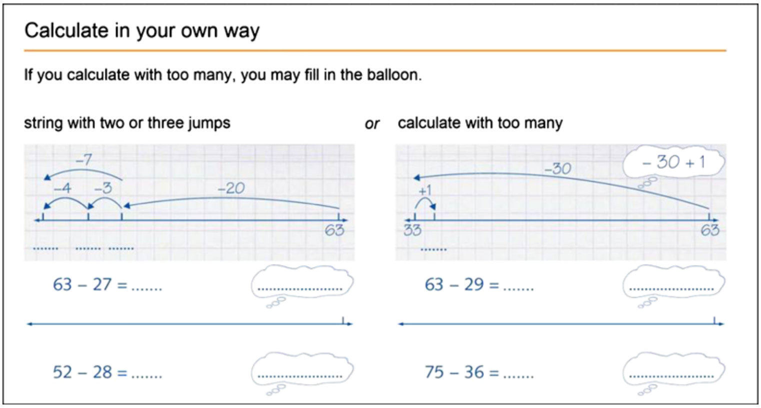

Fig. 8 RR tasks that prompt students to choose an appropriate strategy (RR-book 4b-2, p. 64)

36 weeks of a school year). Both textbooks provide five mathematics lessons each week. The bars in the diagram indicate the number of subtraction tasks per lesson. Every third week in RR and every fourth week in RZ are not filled in (the gray areas). These weeks are meant for evaluation, followed by repetition or enrichment work, and were not included in our analysis.

In RR, the degree of exposure varies: in weeks 1, 7 and 34 relatively more attention is paid to subtraction than in other weeks. In week 1, this concerns the repetition of prerequisite knowledge presented in grade 1, namely number decomposing up to 10. In weeks 7 and 34, a new step in the learning of subtraction is taken. Week 7 is the first time that students encounter subtraction up to 100 and week 34 is the first time that IA is applied to subtraction up to 100. RZ has a fixed pattern of weekly lessons in which 50 to 70 subtraction tasks are offered, with the exception of two periods of three weeks in which almost no attention is paid to subtraction.

Structure of Exposure

Sequence in Types of Problems Table 5a and 5b show how the main types of tasks are distributed over the school year. The gray shading indicates the number of certain types offered: the darker the gray, the larger the number of tasks. The tasks in both textbook series increase in difficulty during the course of the school year. RZ reaches the most difficult types of tasks earlier than RR.

Sequence in Level of Abstraction Both textbook series provide bare number problems, context problems (see Table 2) and tasks with supporting models (see Table 6). However, there is a difference regarding the provided context problems. Both textbook series contain context problems to apply earlier learned subtraction methods (which we consider a performance expectation), but only RR also contains contexts for supporting understanding of subtraction (see Table 6).

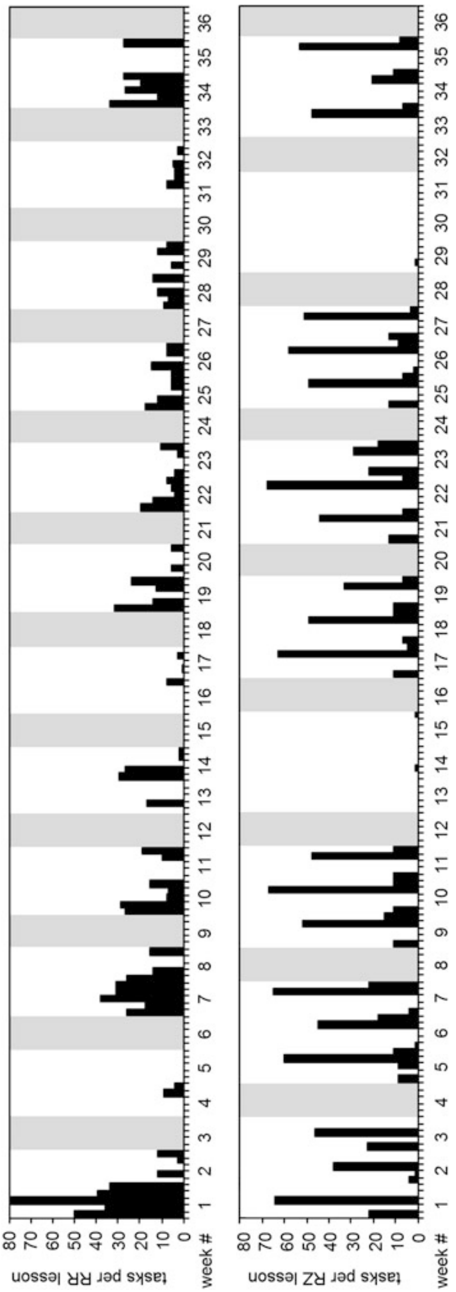

Fig. 9 Distribution of subtraction-related tasks over the school year in RR (*above*) and RZ (*below*)

Table 5a Sequence in types of problems in subtraction-related tasks in RR

Types of problems	Number of RR-tasks								
Subtraction up to 100, bridging a ten					50	54	84	45	145
Subtraction up to 100, without bridging a ten	3		70	49	22	10	31	5	4
Subtraction up to 20	58	173	25	24	23	1	7		
Subtraction up to 10	111	27		12	3				
Prerequisite knowledge	107		21	1	1				
Month #	1	2	3	4	5	6	7	8	9

Table 5b Sequence in types of problems in subtraction-related tasks in RZ

Types of problems	Number of RZ-tasks								
Subtraction up to 100, bridging a ten	4	3	138		25	97	115		98
Subtraction up to 100, without bridging a ten		166	58	1	150	107	88	1	45
Subtraction up to 20	151	57	29	1	13	3	2		5
Subtraction up to 10	39	18	12		9				
Prerequisite knowledge	4					1			
Month #	1	2	3	4	5	6	7	8	9

To get an image of the sequence in level of abstraction, we zoomed in on one particular type of task, namely subtraction up to 20 bridging 10. Figure 10 shows the sequence in level of abstraction of this type of task in the first ten lessons in which it is included. Every black box represents one set of these tasks. Figure 10 illustrates that the sequence in level of abstraction differs between the two textbook series. RR starts with contexts for supporting understanding, followed by tasks with models and then contexts for application. Only in the sixth lesson are bare number tasks provided for the first time. RZ has a different sequence in which bare number tasks and tasks with models are alternated. In contrast with RR, the textbook series RZ begins with bare number tasks. Another difference is that RR provides students with context problems for application several times, while RZ does this only once within the first ten lessons.

Didactical Support in Exposure

Both textbook series offer tasks that provide some form of didactical support. In RR, this is the case in 821 of the total of 1166 subtraction-related tasks (about 70 %) and

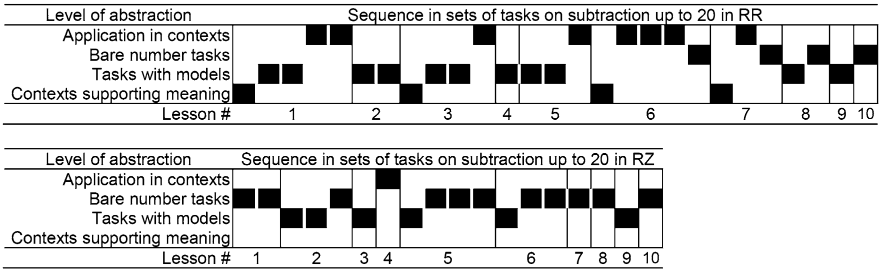

Fig. 10 Sequence in level of abstraction regarding subtraction up to 20 bridging 10 in RR (*above*) and RZ (*below*)

Table 6 Types of didactical support in RR and RZ in grade 2[a]

Didactical support	RR-tasks		RZ-tasks	
	f	%	*f*	%
Use of contexts for supporting understanding	29	4 %	0	0 %
Use of models	423	52 %	108	39 %
Arithmetic rack	102	12 %	0	0 %
Arithmetic blocks	0	0 %	98	35 %
Number line (structured)	11	1 %	10	4 %
Number line (empty)	305	37 %	0	0 %
Number strip	5	1 %	0	0 %
Use of textual instructions	369	45 %	172	61 %
Instructions how to solve the task	186	23 %	108	39 %
Choices offered for solving the task	146	18 %	64	23 %
Reflection-eliciting questions	37	5 %	0	0 %
Total number of tasks with didactical support	821	100 %	280	100 %

[a]See Table 1 note

in RZ, this is the case in 280 of the 1440 subtraction-related tasks (about 19 %) (see Table 6).[9]

Use of Contexts for Supporting Understanding Although both textbook series contain context problems, only in RR do some of the provided contexts serve as a source for new topics to be learned, thus supporting understanding of subtraction (see Table 6). An example is shown in Fig. 11, in which subtracting as adding on (IA) is introduced and related to taking away (DS).

[9]The use of various symbolic representations of subtractions was not included in this count, because by definition every bare number task has some form of symbolic representation.

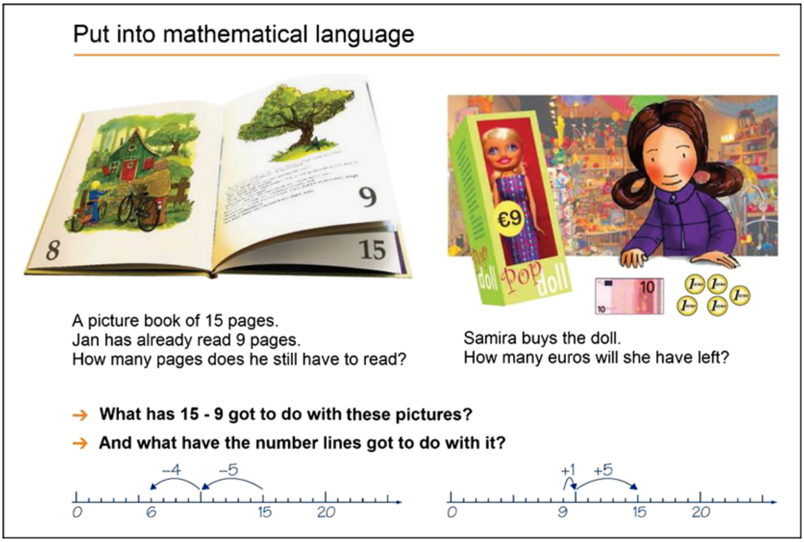

Fig. 11 Relating IA and DS in RR (RR-book 4a, p. 24)

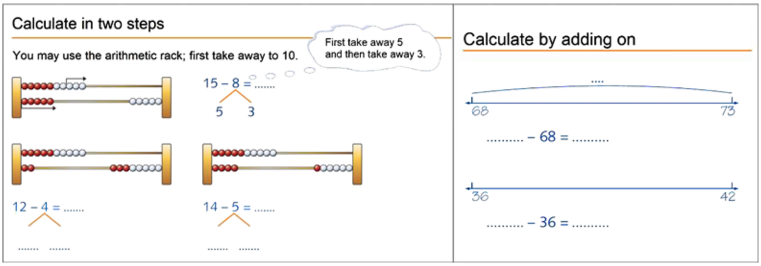

Fig. 12 RR use of the arithmetic rack for subtraction up to 20 (*left*) and the empty number line for subtraction up to 100 (*right*) (RR-book 4a-1, p. 58; 4b-2, p. 78)

Use of Models RR uses the arithmetic rack as the dominant model for subtraction up to 20 and the empty number line for subtraction up to 100 (see Fig. 12). RR uses the empty number line for all calculation methods: stringing combined with DS; IS; IA; and varying (see Fig. 4, Fig. 6, and Fig. 8). In the case of IA, the visualization on the empty number line does not always match the symbolic representation (in 18 of 48 tasks), as can be seen in Fig. 12 (right). In this example, the students are invited to apply an adding on procedure (IA), but the number line (that refers to $73 - \cdots = 68$ or to $68 + \cdots = 73$) and the symbolic representation $\cdots - 68 = \cdots$ do not match to this procedure nor to each other.

RZ uses (pictures of) base-10 arithmetic blocks as the only model for subtraction up to 100 (see Fig. 13). For subtraction up to 20, the structured number line is used also. Although base-10 blocks and the stringing strategy are not epistemologically consistent, RZ uses base-10 blocks as its only supporting model to provide DS combined with stringing, which is the only calculation method that is taught in this textbook series (see section "Carrying out Subtractions"). Furthermore, RZ does not always use this model consistently; sometimes the base-10 structure is not used for subtracting tens (in 11 of 43 tasks, see Fig. 13 [middle]) while at other times it is (in 32 of 43 tasks, see Fig. 13 [right]).

Use of Various Symbolic Representations Besides the standard representation $c - b = \cdots$, both textbook series present little alternative symbolic representations of subtractions. Only RR contains subtraction-related tasks in an addition format (12 of 1166 tasks), to relate subtraction and addition and to elicit subtraction as adding on (IA) (see Fig. 6 [left]). On the other hand, missing number subtractions (e.g., $19 = 20 - \cdots$ and $26 - \cdots = 21$) are only dealt with in RZ (44 of 1440 tasks).

Use of Textual Instructions Both textbooks provide students with textual instructions on how to solve subtractions and offer choices for solving tasks. Reflecting-eliciting questions were only found in RR (see Table 6).

Textual instructions on how to solve subtractions that were found are instructions to use a specific calculation method or how to carry out a specific calculation method. In RR, most of these instructions (120 out of 186) concern subtractions up to 20, and include first subtracting down to 10 and then subtracting the rest (e.g., "First take away to ten", see Fig. 12 [left]). In RZ, most of the instructions (35 out of 108) concern subtractions up to 100, and are about first subtracting the tens and then subtracting the units (e.g. "Step 1: First take away the tens. Step 2: Then take away the units", see Fig. 5 [left]).

Both textbook series offer students choices on how to perform certain tasks. A choice that both offer is whether or not to use a model for solving the task (in RR 53 out of 146 choices offered and in RZ 21 out of 64). The other choices that are offered are rather different in nature. In RR this involves choosing an appropriate calculation method: for instance, to use either a stringing or a varying strategy (see Fig. 8) or to take more or less jumps when using the stringing strategy (in the remaining 93 out of 146 choices offered). In RZ, the remaining 43 (out of 64) choices concern whether or not to use scrap paper.

Questions that prompt students to think and reason about tasks were only found in RR. Examples are: "How did you calculate this?"; and "What has $15 - 9$ got to do with these pictures?" and "And what have the number lines got to do with them?" (see Fig. 11).

Concluding Remarks

Our analysis revealed that freedom of design can result in varying agreement of the potential implemented curriculum with the intended curriculum. In our framework, seven categories—covering content and performance expectations—are related to the intended curriculum. With respect to subtraction up to 100, in three of these

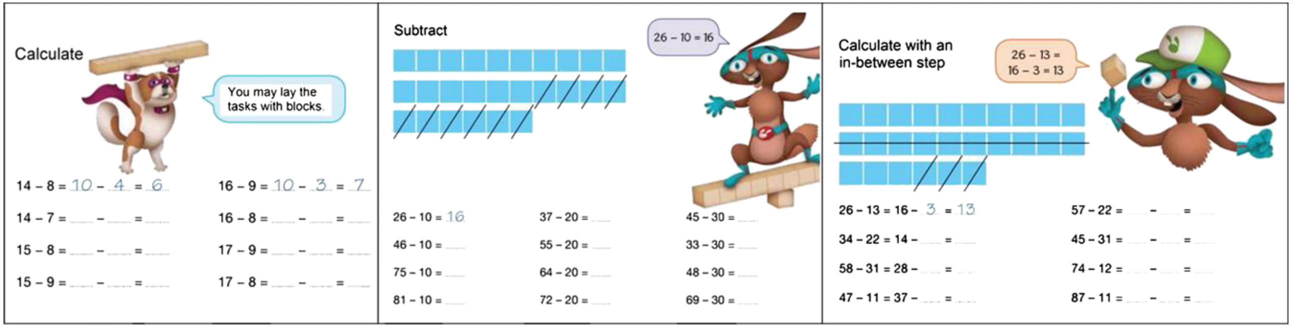

Fig. 13 RZ use of base-10 blocks for subtraction up to 20 (*left*) and up to 100 (*middle* and *right*) (RZ-book 4a, p. 9; 4c p. 25; p. 39)

		Intended curriculum	RR	RZ
Content	Types of problems	Subtraction up to 100	Both RR and RZ offer subtraction up to 100.	
	Format of problems	Context situations	Both RR and RZ offer context situations and bare number problems.	
		Bare number problems	RR offers more context problems than RZ.	RZ offers more bare number problems than RR.
	Semantic structure of problems	Subtraction as taking away	Both RR and RZ present subtraction as taking away.	
		Subtraction as determining the difference	RR presents subtraction as determining the difference.	RZ does not present subtraction as determining the difference.
Performance expectations	Knowing subtraction facts	Knowing subtractions up to 20 by heart	RR puts most emphasis on knowing subtraction facts up to 10.	RZ puts most emphasis on knowing subtraction facts up to 20.
	Carrying out subtractions	Subtraction by standard methods	Both RR and RZ expect students to learn one standard method, namely stringing combined with DS.	
		Subtraction in smart ways	RR expects students to learn IA, IS, and varying strategies.	RZ does not expect students to learn alternative calculation methods.
	Applying subtractions	Using standard methods with insight in real-life situations	Both RR and RZ expect students to apply subtraction methods in context problems.	
			RR offers context situations with various forms and both semantic structures.	RZ offers context situations with one text form and one semantic structure.
	Understanding of subtraction	Understanding of the operation subtraction	RR expects students to explain their thinking, visualize their calculation method and choose appropriate calculation methods.	RZ does not offer clearly distinguishable performance expectations regarding understanding.

Fig. 14 Agreement of RR and RZ with the Dutch intended curriculum regarding subtraction up to 100

categories (types of problems, format of problems and knowing subtraction facts), the textbook series RR and RZ are comparable in their agreement with the Dutch intended curriculum. However, in the other four categories, the fit of RR to the intended curriculum is closer than that of RZ. Figure 14 summarizes our findings.

Regarding the content (research question 1), both textbooks series present subtraction problems up to 100, and both textbook series offer bare number problems as well as context problems. RZ offers more bare number tasks and RR offers more context problems. In deviation of the intended curriculum, RZ only addresses one semantic structure of subtraction. In contrast, RR deals with both.

The degree in which the two textbook series reflect the performance expectations of the intended curriculum (research question 2) also differs. RR offers more tasks on knowing subtractions in total, but RZ presents more tasks on knowing subtractions up to 20. In both textbooks, students are expected to learn the standard calculation method of DS combined with stringing. Only RR expects students to learn alternative calculations methods as well. The way that RZ notates in-between answers can easily lead to incorrect notations (e.g. $12-3=12-2=10-1=9$ instead of $12-3=10-1=9$), especially when students interpret the = symbol

<table>
<tr><th></th><th></th><th>RR</th><th>RZ</th></tr>
<tr><td rowspan="2">Degree of exposure</td><td rowspan="2">Number of tasks</td><td colspan="2">In both RR and RZ about 20% of all tasks in grade 2 addresses subtraction up to 100.</td></tr>
<tr><td>In absolute numbers, RR offers considerably less subtraction-related tasks.</td><td>In absolute numbers, RZ offers considerably more subtraction-related tasks.</td></tr>
<tr><td rowspan="3">Structure of exposure</td><td rowspan="2">Sequence in types of problems</td><td colspan="2">Both RR and RZ have a structure of increasing difficulty in the course of the school year.</td></tr>
<tr><td>RR spends more tasks on prerequisite knowledge.</td><td>RZ reaches more difficult types of tasks at an earlier stage.</td></tr>
<tr><td>Sequence in level of abstraction</td><td>RR uses contexts for supporting understanding as the first step in the sequence of level of abstraction.</td><td>RZ uses bare number tasks as the first step in the sequence of level of abstraction.</td></tr>
<tr><td rowspan="7">Didactical support in exposure</td><td>On the whole</td><td>RR offers considerably more tasks with didactical support.</td><td>RZ offers considerably less tasks with didactical support.</td></tr>
<tr><td>Use of contexts for supporting understanding</td><td>RR uses contexts for supporting understanding of subtraction.</td><td>RZ does not use contexts for supporting understanding of subtraction.</td></tr>
<tr><td>Use of models</td><td>RR uses the empty number line, both for stringing and for alternative methods.
In some tasks, the visualization on the empty number line does not match the symbolic representation.</td><td>RZ uses (pictures of) base-10 blocks for stringing, the only calculation method it offers, even though this model is not epistemologically consistent with this strategy.</td></tr>
<tr><td rowspan="2">Use of various symbolic representations</td><td colspan="2">Both RR and RZ provide the standard symbolic representation c-b=…</td></tr>
<tr><td>RR provides subtractions in addition format (a+…=c).</td><td>RZ provides missing number subtractions (a=c-… and c-…=a).</td></tr>
<tr><td rowspan="2">Use of textual instructions</td><td colspan="2">Both RR and RZ provide textual instructions on how to carry out a specific calculation method.</td></tr>
<tr><td>RR uses textual instructions for choosing an appropriate calculation method and eliciting reflection.</td><td>RZ uses textual instructions for choosing whether or not to use scrap paper.</td></tr>
</table>

Fig. 15 Learning facilitators for subtraction up to 100 in RR and RZ

only as 'results in' and not as an equivalence symbol. Both textbook series employ context problems for application of subtraction, but only in RR is this done by presenting various forms of contexts and by including both semantic structures of subtraction. Finally, only RR contains explicit performance expectations regarding understanding of subtraction.

The two textbook series also differ in the learning facilitators they offer students (research question 3). Figure 15 summarizes our findings on this research question.

RZ offers a larger amount of subtraction-related tasks and reaches more difficult types of tasks at an earlier stage. However, RR spends more tasks on prerequisite knowledge and uses contexts for supporting understanding as the first step in the sequence of level of abstraction, resulting in offering a solid base for the learning of subtraction up to 100. Furthermore, RR offers almost three times as much didactical support compared to RZ. This includes forms of didactical support that are absent in RZ, namely contexts for supporting understanding, textual instructions for choosing appropriate calculation methods, and reflection-eliciting questions. Another shortcoming of RZ is that it uses base-10 arithmetic blocks for supporting stringing, which means that model and strategy are not epistemologically consistent.

To a certain degree, a similar inadequacy applies also to RR when using a particular symbolic representation of subtraction which does not match the presentation on the empty number line. Both examined textbook series do only provide very few tasks involving various symbolic representations of subtraction. The textbook series differ with respect to the textual instructions they provide. RZ offers instructions on how to proceed, whereas RR provides instructions that prompt students to reflect.

Our analysis made it clear that freedom of design can result in a potential curriculum that may deviate from the intended curriculum. The two examined textbook series differ noticeably in their view on subtraction up to 100 as a mathematical topic. RZ reflects a limited view including one semantic structure, one meaning, and one calculation method. RR supports students' development of a broad mental constitution of subtraction, including both meanings and both semantic structures, as well as various calculation methods. Furthermore, our results show that the incorporated ideas of the two textbook series about mathematics education (RR is presented as a RME-oriented textbook series and RZ as an alternative to this approach) actually result in different learning opportunities for students. It really makes a difference for students whether or not they are offered a broad mental constitution of subtraction, whether or not they are given reflection-eliciting questions, and whether or not there is a match between models and symbolic representations or calculation methods.

Of course, what is in the textbook is not necessarily similar to what is taught in class. However, following Valverde et al. (2002, p. 125), we think that "how content is presented in textbooks (with what expectations for performance) is how it will likely be taught in the classroom." Therefore, textbook analysis can provide an inside view in how a subject might be taught. As such, textbook analyses are a crucial tool that can preserve us from having teaching practices not in agreement with the intended curriculum and that do not offer students the desired learning opportunities. How necessary such analyses are was shown when a textbook analysis disclosed that higher-order problem solving is lacking in Dutch mathematics textbooks (Kolovou et al. 2009), even though it is part of the Dutch intended curriculum.

In the present textbook analysis on the topic of subtraction it was again revealed that the textbook matters. The examined textbook series contain different learning opportunities. Disclosing these opportunities is as important as examining the efficacy of textbooks. After all, when students cannot encounter particular content along with sufficient learning facilitators, we cannot expect them to learn this content.

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